IE 518 Discrete Optimization
Take Home Final Examination
April 27, 2001
Due: May 28, 2001

The exam essentially pertains to the problem below.

Assume that you have a set I of clients and a set J of potential sites. You are given a matrix D with entries d_{ij} representing the distance of client i to potential site j. You are allowed to open at most p sites, and you have to assign each client to a an open site. The criterion to minimize is the maximum distance from client i to all the potential sites among all clients. Therefore, we consider the following formulation of the problem. Let binary variables y_j represent whether site j is open or not, and likewise, let x_{ij} be binary variables representing whether client i is assigned to site j. The formulation is

$$\min z \text{ subject to}$$

$$\sum_j x_{ij} = 1 \;\forall\; i \in I$$

$$x_{ij} \leq y_j \;\forall\; i \in I, j \in J$$

$$\sum_j y_j \leq p$$

$$\sum_j d_{ij} x_{ij} \leq z \;\forall\; i \in I$$

This problem is known as the p-**center** problem. Let $F2$ denote the polyhedron described by the equalities and inequalities above.

5 points **1.** Prove that the polyhedron $\mathrm{Co}(F2)$ is full dimensional. (If necessary you may pass to an inequality ($\leq$) description of the first set of constraints by subtracting a constant from all distance coefficients, and modifying the problem accordingly.)

10 points **2.** Prove that the $\sum_j y_j \leq p$ defines a facet of $\mathrm{Co}(F2)$. Are all trivial facet defining inequalities for the p-median problem facet defining for the p-center problem as well? Just sketch proofs.

15 points **3.** Check whether the I. de Farias facet defining inequalities of Theorem 2 in his paper are also valid and/or facet defining for $\mathrm{Co}(F2)$. Can you find other valid and/or facet defining inequalities?

15 points **4.** Propose different Lagrangian Relaxation and Lagrangian Decomposition schemes for this problem. Compare the relative strength of the bounds and their strength vis-a-vis the LP relaxation bound. If you compare the strength of your relaxations computationally, 20 bonus points will be given. The LP relaxation of this problem is known to give quite weak bounds. Therefore, any relaxation scheme that beats the LP relaxation significantly is an important contribution.

25 points **5.** Implement a a Branch and Cut algorithm for the p-center problem. Carefully describe your algorithm and report in a table your computational results in comparison to the CPLEX IP solver. You may carry out your implementation in GAMS, or in AMPL. For examples of how to program loops in GAMS, refer to `www.gams.com` and to the AMPL site. Use the test problems also used by de Farias from the OR Library site by Beasley `http://mscmga.ms.ic.ac.uk/info.html` (Ref. 5 of his paper).

20 points **6.** Propose and implement in a programming language of your choice a primal-dual approximation algorithm for the p-center problem. Report your results on the same set of test problems as in 5 above. Does your algorithm have a performance guarantee (it does not have to, but if it does, 10 bonus points will be given).

10 points **7.** Propose and implement in a programming language of your choice a primal-dual approximation algorithm for the generalized assignment problem. Report your results on the same set of test problems as in 5 above. Does your algorithm have a performance guarantee (it does not have to, but if it does, 10 bonus points will be given).

Instructions

- The exam has seven questions, and a total credit of 100 regular credit points + 40 bonus points.
- Please feel free to use any source you see fit on the condition to provide proper reference to the literature.
- Please copy and sign the following pledge of honour on your exam sheet.

  > I, the undersigned, certify that the answers given below represent the result of my individual work, and that I did not discuss the specifics of any of the questions with my classmates.

- I know it is a lot of work, but can you please type your answers preferably in LaTex?