MATH 520 Linear Algebra
Fall 2005
HW I
22.9.2005 , due: 29.9.2005

Prove or disprove means: either you choose to give a proof or you give a counterexample.

If x and y are different points in $\mathbf{R}^n$, the set of points of the form

$$(1-\lambda)x + \lambda y,\ \lambda \in \mathbf{R}$$

is called the *line through* x and y. A subset M of $\mathbf{R}^n$ is called an *affine set* if $(1-\lambda)x + \lambda y \in M$ for every $x \in M$, $y \in M$ and $\lambda \in \mathbf{R}$. Prove or disprove the following statement:

1. The subspaces of $\mathbf{R}^n$ are affine sets which contain the origin.

For $M \subset \mathbf{R}^n$ and $a \in \mathbf{R}^n$, the *translate* of M is defined to be the set

$$M + a = \{x + a | x \in M\}.$$

Prove or disprove the following statement:

2. The translate of an affine set is another affine set.

An affine set M is said to be parallel to another affine set L if $M = L + a$ for some a. Prove or disprove the following statement:

3. Each non-empty affine set M is parallel to a unique subspace L. This L is given by

$$L = M - M = \{x - y | x \in M, y \in M\}.$$

4. Let V be the vector space of all functions of a variable t. Let $f_1, \ldots, f_n$ be n functions. To say that they are linearly independent is to say there exist n numbers $a_1, \ldots, a_n$ not all equal to 0 such that

$$a_1 f_1(t) + \ldots + a_n f_n(t) = 0$$

for *all* values of t. Prove or disprove:

The two functions e^t and e^{2t} are linearly independent.