## IE 444 Operations Research in Finance Fall 2005

Homework II, due: 6.10.2005

In this homework you will have to write and solve a linear programming model using the XPRESS-MP system via its modeling language MOSEL. Refer to the web site http://www.dash.co.uk for information on the use of XPRESS-MP. After reaching the site click on "Products" and choose "Examples repository". You will find many modeling examples there along with information on how to solve them using the XPRESS-MP IVE environment. The XPRESS-MP system is available in all BCC labs.

- 1. A small company wishes to match obligations over a 6-year period. A universe of 12 bonds with face value equal to \$100 were selected for that purpose, and all accounting is done on a yearly basis. The cash flow structure for each bond is shown in the table below. Below this column is the bond's current price, denoted p. For example the first column represents a 10% bond that matures in 6 years. This bond is selling at 101.5 YTL. The yearly liabilities of the company are 107, 203, 787, 102, 812, and 1204 (in thousand YTL) respectively.
- a. Assuming that no short-term re-investing is possible, formulate a cash flow matching model for this company to minimize the initial cost of the portfolio. Build a MOSEL model and solve this model in XPRESS-MP.
- **b.** Give an interpretation of the optimal values of dual variables associated with the liability matching constraints.
- c. Assume that in-excess cash can be re-invested at a rate of 3.5% per year. Modify your model accordingly. Does your solution change? By how much? For values of the re-investment rate that go from 1% to 4% (take several values in this interval), re-solve the model and plot the cost of your portfolio versus the re-investment rate.
- **d.** Solve the model of c. by maximizing the end-of-horizon excess cash instead of minimizing the cost of the portfolio. How does the solution change?

| year | Bond type |      |      |      |      |      |     |     |     |      |        |       |
|------|-----------|------|------|------|------|------|-----|-----|-----|------|--------|-------|
|      | 1         | 2    | 3    | 4    | 5    | 6    | 7   | 8   | 9   | 10   | 11     | 12    |
| 1    | 10        | 7    | 8    | 5    | 7    | 6    | 10  | 8   | 7   | 100  | 8.25   | 7.5   |
| 2    | 10        | 7    | 8    | 5    | 7    | 6    | 10  | 8   | 107 |      | 8.25   | 7.5   |
| 3    | 10        | 7    | 8    | 5    | 7    | 6    | 110 | 108 |     |      | 108.25 | 7.5   |
| 4    | 10        | 7    | 8    | 5    | 7    | 106  |     |     |     |      |        | 7.5   |
| 5    | 10        | 7    | 8    | 105  | 107  |      |     |     |     |      |        | 107.5 |
| 6    | 110       | 107  | 108  |      |      |      |     |     |     |      |        |       |
| p    | 101.5     | 94.5 | 99.7 | 92.8 | 97.8 | 95.9 | 108 | 104 | 102 | 98.3 | 103    | 98.2  |