MATH 520 Linear Algebra
Fall 2005
HW II
29.9.2005 , due: 6.10.2005

Prove or disprove means: either you choose to give a proof or you give a counterexample.

1. Prove or disprove: If $\{M_\alpha\}$ is a family of affine sets, then so is $\cap_\alpha M_\alpha$.

A point x is said to be an *affine combination* of points $x_1, \dots, x_k$ if there exists a vector $q = (q_1, \dots, q_k)$ in $\mathbf{R}^n$ such that

$$x = q_1 x_1 + \ldots + q_k x_k, \text{ and } \sum_{i=1}^{k} q_i = 1.$$

2. Prove or disprove: A set S is an affine set if and only if every affine combination points in S is in S. (Hint: You may want to use the results of HW1).

A finite list of vectors $x_1, \dots, x_k$ is *affinely dependent* if there exist real numbers $\lambda_1, \dots, \lambda_k$, not all zero such that $\lambda_1 + \ldots + \lambda_k = 0$ and $\lambda_1 x_1 + \ldots + \lambda_k x_k, = 0$. If the list $x_1, \dots, x_k$ is not affinely dependent, it is *affinely independent*.

3. Prove or disprove: A list of vectors $x_1, \dots, x_k$ is *affinely dependent* if and only if the vectors $(x_2 - x_1), (x_3 - x_1), \dots, (x_k - x_1)$ is a linearly dependent list.

4. Is $\{x : Ax = b\} \neq \emptyset$ for $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$ an affine set? Is $\{x : Ax = b, x \geq 0\} \neq \emptyset$ an affine set? Justify your answers.

The dimension of an affine set is defined to be the dimension of the parallel subspace.

5. Prove or disprove: Let M be an affine set. Then the following statements are equivalent:

1. The dimension of M equals r.
2. There exist $r + 1$ vectors $x_0, x_1, \dots, x_r$ in M that are affinely independent, and any list of $r + 2$ vectors in M is affinely dependent.