**IE 303.3 Modeling and Methods in Optimization**
**Fall 2004**
**HW # 1, Due: 30.9.2004**

The homework is expected to be strictly personal work. If you happen to find answers to any questions in a book or on the web, please give proper reference. Failure to do so will result in zero credit.

**1**[20 points] Consider the following problem faced by the manager of an oil refinery. He has 8 million barrels of crude oil A and 5 million barrels of crude oil B. He can use these resources to make either gasoline, with a profit of $4/barrel, or heating oil, with a profit of $3/barrel. He has at his disposal three production processes with the following characteristics. Process 1 uses 3 barrels of crude A and 5 barrels of crude B as input to obtain 4 barrels of gasoline and 3 barrels of heating oil as output. Process 2 needs 1 barrel of crude A and 1 barrel of crude B as input to obtain 1 barrel of gasoline and 1 barrels of heating oil as output. Process 3 uses 5 barrels of crude A and 3 barrels of crude B as input to obtain 3 barrels of gasoline and 4 barrels of heating oil as output. The manager wishes to choose the production levels of these processes to maximize profit. Formulate and solve his problem using MOSEL modeling language and XPRESS-MP software. Attach a copy of your MOSEL model as well as your results.

**2**[40 points] A *fractional programming* problem is one of the form

$$\max_{x \geq 0} z = \frac{c^T x + c_0}{d^T x + d_0}$$

subject to

$$Ax = b$$

where $c$ and $d$ are $n$-vectors, $A$ is $m \times n$ matrix, $b$ is an $m$-vector, $c_0$ and $d_0$ are scalars. It is assumed that $d^T x + d_0 > 0$ for all feasible $x$.

1. [20] Convert this problem into an equivalent LP by defining
$$v(x) = d^T x + d_0 > 0$$
and $y = x/v(x)$, and $y_0 = 1/v(x) > 0$.
2. [20] Let $y^*, y_0^*$ be an optimal solution to this linear programming problem and suppose that $y_0^* \neq 0$. Show that we can use this solution to obtain an optimal solution to the original problem.

**3** [40 points] The infinity-norm of a vector $z \in \mathbf{R}^n$ is defined as

$$\|z\|_\infty = \max_{i=1,\ldots,n} |z_i|.$$

The discrete Tschebyshev approximation problem is defined as: Given a vector $b \in \mathbf{R}^m$ and a linear subspace $L$ of $\mathbf{R^m}$ spanned by vectors $a_1, \ldots, a_n$, find a vector $x$ closest to $b$ in the infinity-norm, i.e., solve the problem

$$\min_x \|b - Ax\|_\infty$$

where $A = [a_1, \ldots, a_n]$ is the $m \times n$ matrix with columns $a_1, \ldots, a_n$.

1. [10] Formulate this problem as linear program.
2. [10] Find the dual linear program.
3. [20] Interpret *geometrically* the dual problem using the original problem data (i.e., $b$, $L$, $a_1, \ldots, a_n$) and the 1-norm of a vector defined as

$$\|q\|_1 = \sum_{i=1}^{m} |q_i|$$

(**Hint:** You may have to define new variables to simplify the dual problem.)