

IE 303.3 Modeling and Methods in Optimization
Fall 2004
HW # 2, Due: 11.10.2004

The homework is expected to be strictly personal work. If you happen to find answers to any questions in a book or on the web, please give proper reference. Failure to do so will result in zero credit.

1[50 points] You are given m identical machines and n jobs. Each job j must be processed for a total of p_j periods. (We assume that each p_j is an integer.) However, preemption is allowed. In other words, the processing of a job can be broken down and can be carried out by different machines in different periods. Each machine can only process one job at a time, and a job can only be processed by a single machine at a time. In addition, each job j is associated with a release time r_j and a deadline d_j : processing cannot start before period r_j , and must be completed before period d_j . Naturally, we assume $r_j + p_j \leq d_j$ for all jobs j . We wish to determine a schedule whereby all jobs are processed, without violating the release times and deadlines, or show that no such schedule exists.

To construct a maximum flow formulation of the problem, we proceed as follows. The first step is to rank all the release times and deadlines in ascending order. The resulting ordered list of numbers divides the time horizon into a number of non-overlapping intervals. Let T_{kl} be the interval that starts in the beginning of period k and ends in the beginning of period l . Note that during each interval T_{kl} , the set of jobs that can be processed does not change. In particular we can process any job j that has been released ($r_j \leq k$) and its deadline has not yet been reached ($l \leq d_j$). For a concrete example suppose that we have four jobs with release times 3, 1, 3, 5 and deadlines 5, 4, 7, 9. The ascending list of release times and deadlines is 1, 3, 4, 5, 7, 9. We then obtain 5 intervals, namely, $T_{13}, T_{34}, T_{45}, T_{57}$, and T_{79} .

To construct the network, we make one source node s , a sink node t , a node corresponding to each job j and a node corresponding to each interval T_{kl} . Your assignment in this question is to define arcs and their capacities, draw the network, interpret flow along an arc and give the complete formulation. Try to solve your formulation with XPRESS-MP using the sample data given in the paragraph above after making up your own processing times.

2[50 points] A catering company must provide to a client r_i tablecloths on each of N consecutive days. The catering company can buy new tablecloths at a price of p dollars each, or launder the used ones. Laundering can be done at a fast service facility that makes the tablecloths unavailable for the next n days and costs f dollars for a tablecloth, or at a slower facility that makes tablecloths unavailable for the next m days (with $m > n$) at a cost of g dollars per tablecloth ($g < f$). The caterer's problem is to decide how to meet the client's demand at minimum cost, starting with no tablecloths and under the assumption that any leftover tablecloths have no value.

- a** Show that the problem can be formulated as a network flow problem (transshipment). *Hint:* Use a node corresponding to clean tablecloths and a node corresponding to dirty tablecloths for each day; more nodes may be needed. Explain your formulation.
- b** Show explicitly the form of the network if $N = 5, n = 1, m = 3$. Make up your own data for the problem parameters and solve the problem in XPRESS-MP.