15.053 To accompany lecture on February 7

- Some additional Linear Programs (not covered in lecture)
  - Airplane Revenue Management
  - Tomotherapy

An Airline Revenue Management Problem

Background: Deregulation occurred in 1978

Prior to Deregulation

- Carriers only allowed to fly certain routes. Hence airlines such as Northwest, Eastern, Southwest, etc.
- Fares determined by Civil Aeronautics Board (CAB) based on mileage and other costs --- (CAB no longer exists)

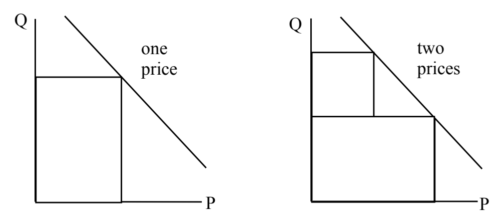
Post Deregulation

- Any carrier can fly anywhere
- Fares determined by carrier (and the market)

Special Features of Airline Economics

- Huge sunk and fixed costs
  - Purchase of airplanes
  - Gate facilities
  - Fuel and crew costs
- Low variable costs per passenger
  - $10/passenger or less on most flights
- Strong economically competitive environment
  - Near-perfect information and negligible cost of information
  - Symmetric information
- No inventories of "product"
  - An empty seat has lost revenue forever: highly *perishable* inventory.

Multiple fare classes: a monopolist's perspective



The two fare model presumes that customers are willing to pay the higher price, even if the lower price is available. How did airlines achieve this?

Two Complexities in Revenue Management

- Complexities due to use of hubs.
  - Many customers transfer airplanes at a hub
  - Hubs permit many more "itineraries" to be flown
- Complexities due to uncertainties
  - Typically the less expensive Q fares are sold in advance of the more expensive Y fares.
  - How many tickets should be reserved for Y fares
- Today: We will focus on the complexities due to hubs, and will consider a very simple example.

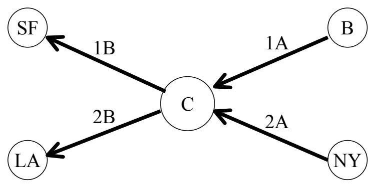
Four Flights from East-West Airlines

Flight #	Depart	Arrive
1A	Boston 8 AM	Chicago 10:15 AM
1B	Chicago 10:45 AM	San Francisco 12:15 PM
2A	New York 7:45 AM	Chicago 10:15 AM
2B	Chicago 10:45 AM	Los Angeles 12:15 PM

Both planes have a seating capacity of 200

Several passenger itineraries can be determined from these flights. For example, a passenger can fly from Boston to Chicago, and another passenger can fly from Boston to LA.

A Diagram Showing the East-West Flights

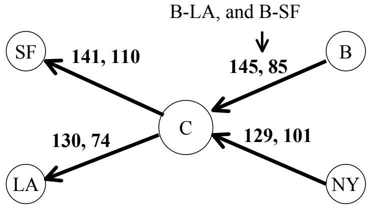


Fares and Demand for Itineraries

Itinerary	Q-class fare and demand		Y-class fare and demand	
B-C	$200	25	$230	20
B-SF	$320	55	$420	40
B-LA	$400	65	$490	25
NY-C	$250	24	$290	16
NY-SF	$410	65	$550	50
NY-LA	$450	40	$550	35
C-SF	$200	21	$230	20
C-LA	$250	25	$300	14

Number of seats allocated if everyone flies

Includes demand from B-C, B-LA, and B-SF



Q-demand, Y-demand

Seat capacity: 200 per flight

Y-fares are higher

Formulation as a Linear Program

- **What are the decision variables?**
- **What is the objective:**
- **What are the constraints?**

An Abstracted version of the LP

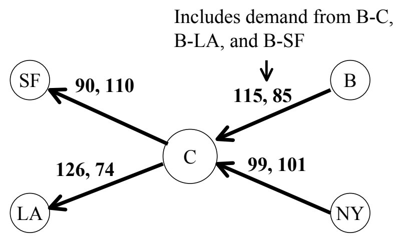
- **Let F be the set of flights**
- **Let C be the set of itineraries/classes**
  - **e.g., <NY-C-SF 7:45-12:15, Q-class> $\in$ C**
- **r_j = revenue from j $\in$ C**
- **d_j = demand for j $\in$ C**
- **let C(f) = subset of C containing flight f**
- **c_f = capacity of flight f**

Work with your partner to formulate the LP

The Optimal Solution

Itinerary	Q-class sold and demand		Y-class sold and demand	
B-C	25	25	20	20
B-SF	25	55	40	40
B-LA	65	65	25	25
NY-C	19	24	16	16
NY-SF	44	65	50	50
NY-LA	36	40	35	35
C-SF	21	21	20	20
C-LA	25	25	14	14

Number of seats allocated in the optimal solution



Q-demand, Y-demand

Seat capacity: 200 per flight

Y-fares are higher

Robert L. Crandall, Chairman, President, and CEO of AMR

I believe that yield management is the single most important technical development in transportation management since we entered the era of airline deregulation in 1979....

The development of American Airline's yield-management system has been long and sometimes difficult, but this investment has paid off. We estimate that yield management has generated $1.4 billion in incremental revenue in the last three years alone. This is not a one-time benefit. We expect it to generate at least $500 million annually for the foreseeable future.

Math Programming and Radiation Therapy

- **Based on notes developed by Rob Freund (with help from Peng Sun)**
- **Lecture notes from 15.094**

Radiation Therapy — Overview

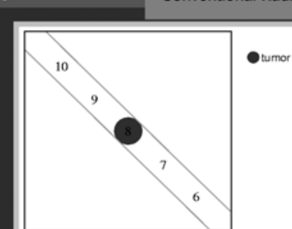
- High doses of radiation (energy/unit mass) can kill cells and/or prevent them from growing and dividing
  - true for cancer cells *and* normal cells
- Radiation is attractive because the repair mechanisms for cancer cells is less efficient than for normal cells

Radiation Therapy — Overview

- Recent advances in radiation therapy now make it possible to:
  - map the cancerous region in greater detail
  - aim a larger number of different "beamlets" with greater specificity
- This has spawned the new field of *tomotherapy*
- "Optimizing the Delivery of Radiation Therapy to Cancer Patients," by Shepard, Ferris, Olivera, and Mackie, *SIAM Review*, Vol. 41, pp. 721–744, 1999.

Radiation Therapy — Overview

Conventional Radiotherapy...

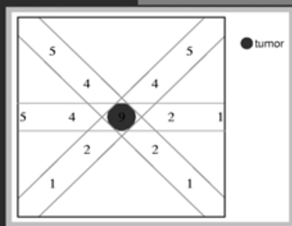


Relative Intensity of Dose Delivered

Radiation Therapy

Overview

...Conventional Radiotherapy...



Relative Intensity of Dose Delivered

Radiation Therapy

Overview

...Conventional Radiotherapy...

In conventional radiotherapy

- 3 to 7 beams of radiation
- radiation oncologist and physicist work together to determine a set of beam angles and beam intensities
- determined by manual "trial-and-error" process

Radiation Therapy

Overview

...Conventional Radiotherapy

With only a small number of beams, it is difficult/impossible to deliver required dose to tumor without impacting the critical area.

Radiation Therapy

Overview

Recent Advances...

- More accurate map of tumor area
  - CT — Computed Tomography
  - MRI — Magnetic Resonance Imaging
- More accurate delivery of radiation
  - IMRT: Intensity Modulated Radiation Therapy
  - Tomotherapy

Radiation Therapy

Overview

...Recent Advances

Radiation Therapy

Overview

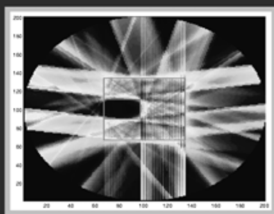
Formal Problem Statement...

- For a given tumor and given critical areas
- For a given set of possible beamlet origins and angles
- Determine the weight on each beamlet such that:
  - dosage over the tumor area will be at least a target level γ_L
  - dosage over the critical area will be at most a target level γ_U

Radiation Therapy

Overview

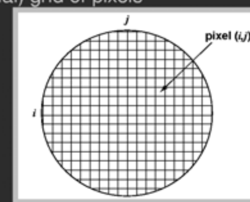
...Formal Problem Statement



Linear Optimization Models

Discretize the Space

Divide up region into a 2-dimensional (or 3-dimensional) grid of pixels



Linear Optimization Models

Create Beamlet Data

Create the beamlet data for each of $p = 1, \ldots, n$ possible beamlets.

D^p is the matrix of unit doses delivered by beam p .

0	0	0	0	0.8	0.8	0	0
0	0	0	0.9	0.9	0	0	0
0	0	0	0.9	0.9	0	0	0
0	0	1.0	1.0	0	0	0	0
0	1.0	1.0	0	0	0	0	0

D^p_{ij} = unit dose delivered to pixel (i,j) by beamlet p.

Linear Optimization Models

Dosage Equations

Decision variables $w = (w_1, \ldots, w_p)$

w_p = intensity weight assigned to beamlet p,

$p = 1, \ldots, n.$

$$D_{ij} := \sum_{p=1}^{n} D^p_{ij} w_p$$

(":=" denotes "by definition")

$$D := \sum_{p=1}^{n} D^p w_p$$

is the matrix of the integral dose (total delivered dose)

Linear Optimization Models

Ideal Linear Model

$$\begin{array}{lll} \underset{w,D}{\text{minimize}} & \sum_{(i,j)} D_{ij} & \\ \text{s.t.} & D_{ij} = \sum_{p=1}^{n} D^p_{ij} w_p & (i,j) \in \mathcal{S} \\ & w \geq 0 & \\ & D_{ij} \geq \gamma_L & (i,j) \in \mathcal{T} \\ & D_{ij} \leq \gamma_U & (i,j) \in \mathcal{C} \end{array}$$

- Unfortunately, this model is typically infeasible.
- Cannot deliver dose to tumor without some harm to critical area(s).

Opportunities for enhancements

- **Use penalties: e.g., $D_{ij} \geq \gamma_L - y_{ij}$ and then penalize y in the objective.**
- **Consider non-linear penalties (e.g., quadratic)**
- **Consider costs that depend on damage rather than on radiation**
- **Develop target doses and penalize deviation from the target**

Linear Optimization Models — Engineered Approaches

$$\underset{w,D}{\text{minimize}} \quad \theta_{\mathcal{T}} \sum_{(i,j)\in\mathcal{T}} D_{ij} + \theta_{\mathcal{C}} \sum_{(i,j)\in\mathcal{C}} D_{ij} + \theta_{\mathcal{N}} \sum_{(i,j)\in\mathcal{N}} D_{ij}$$

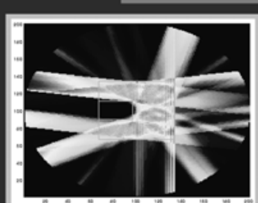
$$\begin{aligned}
\text{s.t.} \quad & D_{ij} = \sum_{p=1}^{n} D_{ij}^{p} w_p \quad && (i,j) \in \mathcal{S} \\
& w \geq 0 && \\
& \gamma_{ij}^{L} \leq D_{ij} \leq \gamma_{ij}^{U} && (i,j) \in \mathcal{T} \\
& w_m \leq \frac{\alpha}{n} \sum_{p=1}^{n} w_p && m = 1, \ldots, p \\
& && (\text{typically } \alpha = 5)
\end{aligned}$$

Computation — Size of the Model — Summary

Variables	Constraints*
63,358	94,191

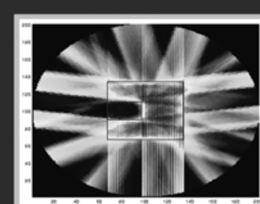
*Excludes variable upper/lower bounds.

Computation — Base Case Model — Optimal Solution



Base Case Model Solution

Computation — Another Model Solution



Solution of a nonlinear model, where $\theta_{\mathcal{N}} = \theta_{\mathcal{C}} = \theta_{\mathcal{T}} = 1$.

Computation — Pre-Processing — ...Heuristics

- Base Case Model
- With Pre-Processing

Code	Algorithm	Iterations	Running Time CPU (sec)	Running Time Wall (minutes)
CPLEX	Simplex	18,428	4.3	4
CPLEX	Barrier	16	130	133

Summary

- **Revenue management, tomotherapy**
- **Models are rarely perfect. One balances the quality of the model with the needs for the situation.**
- **Some techniques used: penalties, reformulations.**