Name:

IE 311.01 Mathematical Programming
Fall 2001
Midterm Examination
22.11.2001

There are 6 questions and the weights add up to 20.

1.[4 points] Consider the dilemma of the New Horizons Student Club. The New Horizons members are trying to decide what fund-raising projects to undertake at the next university fair. One option is customized T-shirts, which will sell for $20 each. The other option is sweatshirts selling for $30. Historical records show that everything offered for sale will be sold before the fair is over. Materials to make the shirts are all donated by Ankuva merchants, but the Student Club must rent the equipment for customization. Different processes are involved, with the T-shirt equipment renting at $550 for the period up to the fair, and the sweatshirt equipment for $720. Display space presents another consideration. The Club has only 300 square feet of display wall area at the fair, and T-shirts will consume 1.5 square feet each, sweatshirts 4 square feet each. The members wish to know what plan will result in the largest income for the Club. Formulate the problem as a mixed-integer linear program. Do not solve it.

2.[3 points] Consider the New Horizons Student Club problem of Question 1. Propose a Lagrangian relaxation scheme where you keep the display area constraint in the problem and dualize the other constraints. Do you recognize the Lagrangian subproblem? Do you expect to get better bounds than the LP relaxation bound or not? Why?

3. [4 points]
Consider the following IP:

$$\max -7x_1 - 3x_2 - 4x_3 \tag{1}$$

subject to

$$x_1 + 2x_2 + 3x_3 - x_4 = 8 \tag{2}$$

$$3x_1 + x_2 + x_3 - x_5 = 5 \tag{3}$$

where all variables are required to be integer valued and nonnegative. The following branch and bound tree has been obtained while solving the problem. Identify the optimal solution from the tree. (You need the give the optimal values of the variables.)

4 [4 points]. Consider the IP:

$$\min 10x_1 + 20x_2 + 40x_3 + 80x_4 - 144y \tag{4}$$

subject to

$$x_1 + x_2 + x_3 + x_4 \geq 4y \tag{5}$$

where all variables x_1, x_2, x_3, x_4, y are binary valued. Solve the IP problem by inspection. Solve the LP relaxation by inspection. Propose an equivalent formulation (using more constraints if necessary) with a stronger LP relaxation.

5.[2 points] Consider the IP

$$\max 40x_1 + 5x_2 + 60x_3 + 8x_4 \tag{6}$$

subject to

$$18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25 \tag{7}$$

where all variables x_1, x_2, x_3, x_4 are binary valued. The LP relaxation optimum is $(5/18, 0, 1, 0)$. Propose a valid inequality that will cut off the LP relaxation optimum.

6.[3 points] Given a connected graph $G = (V, E)$ with node set V and edge set E, and non-negative edge weights c_e for all $e \in E$, the minimum weight spanning tree problem is to find a connected, acyclic subgraph of G containing all the nodes of the graph. Alternatively, (assuming that G has m nodes), a spanning tree is an acyclic subgraph of G containing all the nodes of G, and exactly $m - 1$ edges of G. Give an integer programming formulation of the problem of finding a minimum weight spanning tree in a graph. (Hint: Recall that in class we have seen an integer programming formulation for the **maximum** weight spanning tree formulation. You are welcome to make use of that formulation.)