IE 303.3 Modeling and Methods in Optimization
Fall 2004
Midterm Examination, 18.11.2004

You have 2 hours for five problems. When in doubt, make your assumptions, state them clearly, and solve the problem accordingly. Absolutely no questions! Closed book and notes.

1[20 points] Military commanders are planning the command structure for 6 new radar stations. Three commanders will each be in charge of two of the stations. The following table shows the projected cost (in millions of dollars) of building the necessary communication links to connect jointly commanded locations.

	2	3	4	5	6
1	42	65	29	31	55
2	-	20	39	40	21
3	-	-	68	55	22
4	-	-	-	30	39
5	-	-	-	-	47

Planners seek a minimum cost way to organize the command. Formulate the problem as a graph optimization problem. Identify the problem. Add the necessary constraints to make sure it can be solved as a linear program.

2[30 points] MMM Mills has 800 thousand, 740 thousand, and 460 thousand bushels of corn stored at its three rural operators. Its three processing plants will soon require 220 thousand, 1060 thousand and 720 thousand, respectively to make cornstarch. The following table shows the cost per thousand bushels of shipping from each operator to each plant.

Elevator	Plant		
	1	2	3
1	11	14	21
2	14	10	12
3	16	15	19

MMM wants to move its corn to plants at minimum total shipping cost.

a Formulate a network optimization problem to choose an optimal shipping plan. Identify the problem clearly.

b Find the dual problem.

c Using the primal and dual problems, find an upper bound and a lower bound to the optimal value.

3[20 points] The following table shows the available links and linking costs joining 6 points on the plane. An engineer wishes to find the shortest route visiting all nodes exactly one and using only links shown with a cost figure in the table.

	1	2	3	4	5	6
1	-	10	1	-	1	-
2	10	-	-	1	-	1
3	1	-	-	10	1	-
4	-	1	10	-	-	1
5	1	-	1	-	-	10
6	-	1	-	1	10	-

Formulate this problem as one of the graph optimization problems discussed in class. Identify the problem clearly. Does the problem require subtour elimination constraints? If you did not use the subtour elimination constraints at all, what would be the optimal solution?

4[20 points] Consider the objective function

$$\min \theta_1(x_1) + \theta_2(x_2)$$

where

$$\theta_1(x_1) = \begin{cases} 140 + 8x_1 & \text{if} x_1 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\theta_2(x_2) = \begin{cases} 130 + 9x_2 & \text{if} x_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

with the following constraints on x_1, x_2:

$$x_1 + x_2 \geq 8, 0 \leq x_1 \leq 3, 0 \leq x_2 \leq 8.$$

Form a corresponding integer programming model using binary variables.

5[10 points] Consider the following integer linear program:

$$\max 8x_1 + 3x_2 + 15x_3 + 7x_4 + 10x_5 + 12x_6$$

subject to

$$10x_1 + 6x_2 + 23x_2 + 11x_4 + 10x_5 + 31x_6 \leq 36$$

where all variables are required to be binary. Solve the LP relaxation of the above problem without using the simplex method. Explain your reasoning.