IE 303.3 Modeling and Methods in Optimization
Fall 2004
Second Examination, 23.12.2004

You have 2 hours for five problems. When in doubt, make your assumptions, state them clearly, and solve the problem accordingly. Absolutely no questions! Closed book and notes.

1[20 points] The AAA advertising company manages the television promotion of a variety of products. In the next few months they are planning to cross-advertise six of their items by running interlocking television ads that mention both products. The following table shows AAA's estimates of the number of viewers (in millions) who might be interested jointly in each pair of products.

	2	3	4	5	6
1	7	8	9	11	5
2	-	20	16	14	11
3	-	-	16	15	12
4	-	-	-	13	9
5	-	-	-	-	7

AAA seeks a product pairing that maximizes the appeal, with each product in exactly one pair. Formulate the problem as a graph optimization problem. Identify the problem. Add the necessary constraints to make sure it can be solved as a linear program.

2[30 points] An oil company has 5 platforms drilling off in the Gulf Coast of the United States. The following table shows the east-west and north-south coordinates of their shore base at point 0 and all the platform locations.

Elevator	Platforms					
0	1	2	3	4	5	
E/W	80	10	60	30	85	15
N/S	95	15	70	10	75	30

Each day a helicopter delivers supplies by flying from the base to all platforms and then returning to base.

a Compute the rectilinear distance between each platform and the base and among the platforms. For example the rectilinear distance between base and platform 1 is $(80-10)+(95-15) = 150$.

b Formulate an optimization problem to choose an optimal shortest flying plan using rectilinear distances. Identify the problem clearly. Give all the necessary constraints.

3[20 points] Consider the following integer program:

$$\max x_1 + x_2 + x_3$$

subject to

$$x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

$$x_2 + x_3 \leq 1$$

with all variables binary. Find the optimal solution to the LP relaxation. Is this a lower bound to the optimal value or an upper bound? Add a valid inequality that cuts off this LP relaxation optimum without affecting the integer feasible points. After adding the inequality if you solve the LP relaxation again, does the objective function value increase or decrease?

4[10 points] Consider the objective function

$$\min \theta_1(x_1) + \theta_2(x_2)$$

where

$$\theta_1(x_1) = \begin{cases} 10 + 7x_1 & \text{if} x_1 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\theta_2(x_2) = \begin{cases} 13 + 5x_2 & \text{if} x_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

with the following constraints on x_1, x_2:

$$x_1 + x_2 \geq 5, 0 \leq x_1 \leq 3, 0 \leq x_2 \leq 5.$$

Form a corresponding integer programming model using binary variables.

5[20 points] Solve the following $0-1$ problem by Branch & Bound:

$$\min 90x_1 + 50x_2 + 54x_3 + 60x_4 + 56x_5$$

subject to

$$60x_1 + 110x_2 + 150x_3 + 88x_4 + 70x_5 \geq 150$$

(with all variables binary). Show all your work.