Name:

MATH 520 Linear Algebra
Fall 2002
Midterm Examination I
23.10.2002

There are 8 questions and the weights add up to 20. Prove or disprove means: either you choose to give a proof or you give a counterexample. Please answer the questions in the space provided.

1.[3 points] Prove or disprove the following statement:

> Let $v_1, \ldots, v_n$ be elements of a vector space V. Let W be the set of all linear combinations of $v_1, \ldots, v_n$. Then W is a subspace of V.

2.[2 points] Let V be the vector space of all functions of a variable t. Let $f_1, \ldots, f_n$ be n functions. To say that they are linearly independent is to say there exist n numbers $a_1, \ldots, a_n$ not all equal to 0 such that

$$a_1 f_1(t) + \ldots + a_n f_n(t) = 0$$

for *all* values of t. Prove or disprove:

The two functions e^t and e^{2t} are linearly independent.

3. [3 points]
Prove or disprove the following statement:

> Let V be a vector space. Let $v_1, \ldots, v_n$ be linearly independent elements of V. Let $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ be numbers. Suppose that we have
>
> $$x_1 v_1 + \ldots + x_n v_n = y_1 v_1 + \ldots + y_n v_n.$$
>
> Then we have $x_i = y_i$ for $i = 1, \ldots, n$.

4. [2 points] Prove or disprove:

Let V be a vector space, and let W be a subspace of V. If $\dim W = \dim V$ then $V = W$.

5.[3 points] Prove or disprove:

Let V be a finite dimensional vector space over the field F. Let W be a subpace. Then there exists a subspace U such that V is the direct sum of W and U.

6.[2 points] Prove or disprove:

The vectors $(1,1)$ and $(-3,2)$ in $\mathbf{R}^2$ are linearly independent.

7.[3 points] Which of the following functions are injective? Justify your answer.

a. $f(x) = x^2$
b. $f(x) = \sin x$
c. $f(x) = x + 1$.

8.[2 points] Prove or disprove:

Let $T : V \mapsto W$ be a linear map. Let u, v be elements of V, and let $Tu = w$. If $Tv = 0$, then $T(u+v) = w$.