Name:

MATH 520 Linear Algebra
Fall 2002
Examination II
20.11.2002

There are 8 questions and the weights add up to 20. Prove or disprove means: either you choose to give a proof or you give a counterexample. Please answer the questions in the space provided.

1.[2 points] Prove or disprove the following statement:

$T \in \mathcal{L}(\Re^2)$ has a matrix given as:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

The eigenvectors of T generate a 2-dimensional space.

2.[2 points] Prove or disprove the following statement:

$T \in \mathcal{L}(\Re^2)$ has a matrix given as:

$$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}.$$

where $a \in F$. The eigenvectors of T generate a 1-dimensional space.

3. [3 points]
Prove or disprove the following statement:

> If v_1 and v_2 are eigenvectors of $T \in \mathcal{L}(V)$ with different eigenvalues $\lambda_1 \neq \lambda_2$, then $v_1 + v_2$ is also an eigenvector of T.

4. [2 points] Prove or disprove:

> Let V be a vector space and let $T \in \mathcal{L}(V)$. Let $\lambda \in F$. Let V_λ be the subspace of V generated by all eigenvectors of T having λ as eigenvalue. Then, every nonzero element V_λ is an eigenvector of T having λ as eigenvalue.

5.[3 points] Prove or disprove:

Let $T : U \mapsto V$ be a linear map, and assume that this map has an inverse map $G : V \mapsto U$. Then G is alinear map.

6.[2 points] Prove or disprove:

Let $T : U \mapsto V$ be a linear map whose kernel (null space) equals $\{0\}$, and which is surjective. Then T has an inverse linear map.

7.[3 points] Prove or disprove:

Let V be a vector space of dimension n. Let $\{v_1, \ldots, v_n\}$ be a basis for v. Let $L : \Re^n \mapsto V$ be the map such that

$$L(x_1, \ldots, x_n) = x_1 v_1 + \ldots + x_n v_n.$$

Then L is an isomorphism.

8.[3 points] Prove or disprove:

Let $L : V \mapsto W$ be a linear map. Assume that $dim V = dim W$. If $null L = \{0\}$ then L is invertible.