

## **NONNEGATIVE WEALTH, OPTIONS, AND C-CAPM**

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According to Black-Scholes option pricing model, options are redundant securities having no allocational role in the economy. This article develops a model where agents face nonnegative wealth constraints in a multi-period securities market. The optimal consumption for the representative agent leads to a multifactor conditional C-CAPM, where option returns appear as factors. The empirical tests carried for the period 1990-2006 reveal that at-the-money calls and out-of-the-money puts written on S&P 500 index are priced risk factors. The findings have implications for asset pricing, and allocational role of options.

This article proposes to solve individuals' consumption-investment problem with nonnegative wealth constraints in a multi-period securities market framework, and subsequently derive the optimal sharing rules for agents in an economy where there is the possibility of trading time-event contingent claims. The derivation of optimal sharing rules in equilibrium yields a multifactor conditional consumption capital asset pricing model (C-CAPM), where the first factor is the change in log aggregate consumption, and other factors being the excess returns of a bundle of options written on the aggregate consumption. The empirical tests carried for the

period 1990-2006 document that S&P 500 at-the-money call and out-of-the-money put options appear as priced risk factors both in conditional and unconditional versions of C-CAPM. To the best of our knowledge, this is the first study that documents the significance of option returns in a C-CAPM framework.

There are three important lines of literature that sets the motivating ground for this article. These are:

- i) Empirical tests of single factor asset pricing models - why do CAPM and C-CAPM fail to empirically explain asset prices although they have sound theoretical backgrounds?
- ii) Market completeness and allocational role of options in the economy - what are the possible frictions that lead to incomplete markets, and do these frictions lead options to play an allocational role in the economy?
- iii) Nonnegative wealth constraints - what are the pricing consequences of nonnegative wealth constraints?

Today, there is a vast amount of literature documenting the failure of CAPM and C-CAPM in explaining the cross-section of returns.<sup>1</sup> This is largely explained by two lines of reasoning. Either, multiple factor models are needed as in Merton's I-CAPM (1973), or existing models cannot capture the possible time variation in securities returns. Furthermore, it is documented that conditional and multifactor models fare better in explaining the cross-section of securities returns compared to their unconditional and single factor counterparts.<sup>2</sup>

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<sup>1</sup> See Basu (1977), Banz (1981), Reinganum (1981), and Rosenberg, Reid, and Lanstein (1984) for empirical tests of CAPM, and Hansen and Singleton (1982, 1983), Mankiw and Shapiro (1986), Breeden, Gibbons, and Litzenberger (1989), Campbell (1996), Cochrane (1996), Hodrick, Ng and Sengmueller (1999) for tests of C-CAPM.

<sup>2</sup> See Chen, Roll and Ross (1984), Fama and French (1992, 1993) for tests of multifactor asset pricing models; Jagannathan and Wang (1996) for tests of conditional CAPM; and Lettau and Ludvigson (2001b) for tests of conditional C-CAPM.

Another line of research that has received extensive attention is on the spanning role of options and market completeness. Although standard Black-Scholes option pricing framework is still widely used in practice, research today has shifted rather from assuming complete markets to the examination of the possible factors that lead to incomplete markets. The literature has identified heterogeneous beliefs of agents, asymmetric information, stochastic volatility and jumps, transaction costs, and limitations on short sales or borrowing as possible factors. When these factors are considered, options might become non-redundant, and play an allocational role in the economy.

Research regarding heterogeneous beliefs argues that heterogeneous attitudes towards risk can generate demand for options. Grossmann and Zhou (1996) show that if one of the agents, such as a portfolio insurer, is infinitely averse to the risk when his wealth drops below a given threshold, then the demand for options can be an important determinant of the underlying asset price. Bates (2001) considers an economy where crashes can occur and less crash-tolerant investors buy options from more crash-tolerant ones. In his setting, options complete the market by serving as a hedge against crash risk. Buraschi and Jiltsov (2003) consider a symmetric but incomplete information setting; agents agree on the dividend process but differ in their beliefs about the price process unrelated to fundamentals. They find that much of the observed option trading volume can be explained by this heterogeneity in beliefs.

A number of studies suggest that options might be non-redundant, because the price of a traded option can convey some information, which otherwise would be unobservable in the economy. For example, Grossman (1988) argues that an option

can be non-redundant due to its informational content, thus its removal from the economy would make markets incomplete. Back (1993) shows that, in a market with asymmetric information, the introduction of options might change the stochastic pricing process of the underlying asset. Hence, options introduced to a complete market may be non-redundant. Also Easley, O'Hara, and Srinivas (1998) suggest that an option market could be a platform for informed trading due to lower transaction costs and greater financial leverage.

Furthermore, the presence of stochastic volatility and jumps can severely affect asset price dynamics and thus options that are written on them. The main approach to modeling stock returns is to define a continuous time stochastic volatility diffusion process possibly augmented with an independent jump process in returns. Today, most option pricing models incorporate these two factors in order to account for a more realistic pricing process.<sup>3</sup> Bakshi, Cao, and Chen (1997) compare empirical performances of these alternative option pricing models and conclude that models that include stochastic volatility and jump processes performs better.

Besides these theoretical models, recently, a number of empirical papers have demonstrated that options are non-redundant. Buraschi and Jackwerth (2001) suggest that option returns do significantly increase the spanning quality of the pricing kernel and argue that the volatility risk might be priced in options market. Furthermore, Coval and Shumway (2001) give preliminary evidence that at-the-money straddles can account for the systematic volatility risk in the securities market. Bakshi and

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<sup>3</sup> See Heston (1993) who proposed a stochastic volatility diffusion model, for which one could analytically derive an option pricing formula. Duffie and Kan (1996), and Duffie, Pan, and Singleton (2000) further developed Heston's model to a rich class of affine jump diffusion processes. Several other authors have used stochastic volatility diffusion process augmented by jumps [Bates (1996) Andersen, Benzoni and Lund (2001), Eraker, Johannes and Polson (2001), Pan (2002), Chernov, Gallant, Ghysels and Tauchen (2003)].

Kapadia (2003) show that delta-hedged option portfolios consistently earn negative returns, indicating that there exists a negative volatility risk premium in option prices. Liu and Pan (2003) show that at-the-money straddles, and out-of-the-money puts can be used to complete the markets when markets are incomplete due to volatility and jump risks. Finally, Arısoy, Salih, and Akdeniz (2007) show that S&P 500 straddle returns play an important role in constructing the stochastic discount factor.

The standard asset pricing and option pricing theories assume that markets are frictionless. However, the presence of transaction costs, portfolio constraints such as constraints on short selling, or credit constraints such as nonnegative wealth constraints can generate demand for options. For example, Lee and Yi (2001) find that options with lower transaction costs attract more informed investors. Furthermore, Başak and Croitoru (2000), show that a mispricing between a stock and a redundant derivative arises due to portfolio constraints on short selling and investors with heterogeneous beliefs.

There are other imperfections in the market that can cause agents fail to replicate their consumption patterns via existing securities. One such imperfection is the bounded-credit assumption put forward by Dybvig and Huang (1991). Classical asset pricing theories assume no restrictions on borrowing and lending. However, in real life borrowing is limited due to agents' possibility to default. In their model agents can borrow at most the amount equal to their wealth i.e. no guarantees on future income is allowed, allowing agents to come back to the economy with the ability to pay off their debts. This also rules out doubling strategies and arbitrage possibilities posit by Harrison and Kreps (1979).

Recently, Vanden (2004) analyzes the effects of nonnegative wealth constraints and finds that incorporation of such real-life frictions on wealth implies non-redundancy of options in a single period setting. It is argued that options can effectively complete markets via their leverage properties, i.e. by their limited liabilities and theoretically unlimited payoffs and help agents avoid insolvency and thus meet the nonnegative wealth constraint. This difference in the payoff pattern of options help agents to match their consumption patterns better compared to the limited spanning capacity of existing securities in the economy. In Vanden (2004), single period and continuous time equilibrium properties of nonnegative wealth constraints on agents' consumption-investment problem have been derived, however this problem has not yet been analyzed in a multi-period securities market context.

Setting the problem in a multi-period framework is more appealing, since the real-life practice is to trade securities through dynamically managed portfolios. In this setting, agents have the possibility to trade time-event contingent claims. The contribution of this article will be to set up the problem in a multi-period framework with the introduction of short-lived options, i.e. that can be traded in smaller intervals of time (from a day to up to a year), and correspondingly develop an asset pricing model. In equilibrium, the pricing agent's optimal consumption incorporates the aggregate consumption, and a bundle of short-lived options written on the aggregate consumption with different strike prices. This result is similar to Vanden's, yet the asset pricing consequences are different. Since agents dynamically rebalance their portfolios at each period, we obtain a multifactor conditional C-CAPM model that is empirically testable. To the best of our knowledge, there have not been any multifactor empirical tests of asset pricing that combines C-CAPM framework with

option returns. Thus, the article has implications for asset pricing, portfolio management, and capital markets theories.

The rest of the article is organized as follows. Section 1 introduces the problem, and derives the corresponding asset pricing model. Section 2 draws the econometric framework for tests of the model. Section 3 presents the empirical findings, and Section 4 offers concluding remarks.

## 1 THE MODEL

There are  $I$  agents in the economy indexed by  $i = 1, 2, \dots, I$ . Agents live in a multiperiod pure exchange economy ( $t = 0, 1, \dots, T$ ) with reconvening markets, and agree on the possibilities of occurrences of events in the economy. Each event,  $a_t$ , is a collection of states,  $\omega$ .  $\Omega$  denotes the collection of all possible states of nature, and the true state of the nature is partially revealed to individuals over time.

There is a single perishable consumption good available for consumption at each trading date. Individuals are endowed with nonnegative time-0 consumption, and time-event contingent claims  $\{e_i(0), e_i(a_t), a_t \in F_t; t = 1, \dots, T\}$ , and have the possibility to trade these claims after  $t = 0$ .<sup>4</sup>

Before proceeding with the model we have to note some simplifying assumptions. We assume that i) all agents have the same information structure,  $F_t$ , ii) all agents agree on the possible states of nature, iii) all agents are endowed with an initial wealth, iv) all agents have time-additive, and state independent von Neumann-

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<sup>4</sup> A time-event contingent claim is a security that pays one unit of consumption at a trading date  $t \geq 1$  in an event  $a_t \in F_t$  and nothing otherwise. The notation and the setting follow the desktop reference Huang and Litzenberger (1988).

Morgenstern utility functions with identical cautiousness, v) all agents face nonnegative wealth constraints at all periods, and vi) markets are dynamically incomplete. Under these assumptions, the model proceeds as follows.

Individual  $i$  has preferences for time-0 consumption, and time-event contingent claims that are increasing, strictly concave, and differentiable, i.e.

$u_{i0}(z_i(o)) + \sum_{t=1}^T \sum_{a_t \in F_t} p_{a_t} u_{i,t}(z_i(a_t))$ .  $p_{a_t}$  is the homogeneously agreed probability of the occurrence of the event  $a_t \in F_t$ ,  $z(0)$  is the time-0 consumption good, and  $z(a_t)$  are the payoffs of the time-event contingent claims in the event  $a_t \in F_t$  for  $t \geq 1$ , respectively.

Each agent tries to maximize their expected utilities over their lifetime while facing nonnegative wealth and budget constraints. Now, define  $\phi(0)$  as the price of the time-0 consumption good, and  $\phi_{a_s}(a_t)$  as the ex-dividend price for the time-event contingent claim paying off at time  $s$  in event  $a_s$ , conditional on the occurrence of event  $a_t$  at time  $t$ , where

$$\phi_{a_s}(a_t) = \begin{cases} \frac{\phi_{a_s}}{\phi_{a_t}} & \text{if } t < s \text{ and } a_s \subseteq a_t \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Also, for  $t < s$ , define  $p_{a_s}(a_t)$  to be the conditional probability of event  $a_s$  given that at time  $t$  event  $a_t$  occurs, so that

$$p_{a_s}(a_t) = \begin{cases} 0 & \text{if } a_s \not\subseteq a_t \\ \frac{p_{a_s}}{p_{a_t}} & \text{if } a_s \subseteq a_t \end{cases} \quad (2)$$

With the above assumptions, the problem can be formulated as follows:

$$\begin{aligned}
& \underset{\{z_i(0), z_i(a_t)\}}{\text{Max}} \quad u_{i,0}(z_i(0)) + \sum_{t=1}^T \sum_{\substack{a_s \in F_s \\ a_s \subseteq a_t}} p_{a_s}(a_t) u_{i,s}(z_i(a_s)) \\
& \text{s.t.} \quad \phi_0 z_i(0) + \sum_{t=1}^T \sum_{\substack{a_s \in F_s \\ a_s \subseteq a_t}} \phi_{a_s}(a_t) z_i(a_s) = \phi_0 e_i(0) + \sum_{t=1}^T \sum_{\substack{a_s \in F_s \\ a_s \subseteq a_t}} \phi_{a_s}(a_t) e_i(a_s) \\
& \quad z_i(0) \geq 0 \\
& \quad z_i(a_s) \geq 0 \quad \forall a_s \in F_s, a_s \subseteq a_t
\end{aligned}$$

The solution to the above problem requires the solution of Lagrangian and its associated Kuhn-Tucker conditions. Before proceeding with the solution, we show that when  $t=1$ , the optimal consumptions are equivalent to Vanden's optimal sharing rules derived in the single period setting, and the marginal rates of substitutions are equal across agents.

**Proposition 1.** *The solution of the above problem in the single period setting implies that the marginal rates of substitutions of agents are equal for all  $i=1, \dots, I$ , and the optimal consumption at  $t=0$ , and  $t=1$  for individual  $i$  is given by*

$$c_i(0) = \max \left[ 0, u'_{i,0}(\gamma_{i,0} \phi_0) \right], \text{ and } c_i(a_1) = \max \left[ 0, u'_{i,0} \left( \frac{\gamma_{i,0} \phi_{a_1}(0)}{p_{a_1}(0)} \right) \right].$$

**Proof.** The Lagrangian for the single period version can be written as;

$$L(z_i(0), z_i(a_1)) = u_{i,0}(z_i(0)) + \sum_{\{a_1 \in F_1\}} p_{a_1}(0) u_{i,1}(z_i(a_1))$$

$$\begin{aligned}
& +\gamma_{i,0} \left( \phi_0 [e_i(0) - z_i(0)] + \sum_{\{a_1 \in F_1\}} \phi_{a_1}(0) [e_i(a_1) - z_i(a_1)] \right) \\
& + \mu_{i,0} z_i(0) + \mu_{i,a_1} z_i(a_1)
\end{aligned}$$

The first order conditions (F.O.C.) for the above Lagrangian are evaluated at  $c_i(0)$  and  $c_i(a_1)$ , since the only sources of consumption are the payoffs of time-event contingent claims,  $z_i(0)$  and  $z_i(a_1)$ ;

$$\mathbf{c_i(0):} \quad u'_{i,0}(c_i(0)) - \gamma_{i,0} \phi_0 = 0 \quad (3)$$

$$\mathbf{c_i(a_1):} \quad p_{a_1}(0) u'_{i,1}(c_i(a_1)) - \gamma_{i,0} \phi_{a_1}(0) = 0 \quad (4)$$

$$\mathbf{K-T:} \quad \mu_{i,0} c_i(0) = 0 \quad (5)$$

$$\mu_{i,a_1} c_i(a_1) = 0 \quad (6)$$

From Kuhn-Tucker (K-T) conditions (5) and (6), when the nonnegative constraints do not bind, i.e.  $c_i(0) > 0$ , and  $c_i(a_1) > 0$ , we have  $\mu_{i,0} = \mu_{i,a_1} = 0$ . If the nonnegative constraints bind, then wealth at each period and correspondingly consumption at each period is zero. Thus, the above problem has a solution at either zero consumption (when nonnegative constraints bind), or some positive levels of  $c_i(0)$ , and  $c_i(a_1)$  that is evaluated at  $\mu_{i,0} = \mu_{i,a_1} = 0$ .

Thus,  $\frac{p_{a_1}(0) u'_{i,1}(c_i(a_1))}{u'_{i,0}(c_i(0))} = \frac{\phi_{a_1}(0)}{\phi_0}$ . In other words, the marginal rate of

substitutions of agents in the economy are equal, and independent of the index  $i$ . Also, the optimal time-0, and time-1 event  $a_1$  consumptions are given by

$c_i(0) = \max \left[ 0, u'_{i,0}(\gamma_{i,0}\phi_0) \right]$ , and  $c_i(a_1) = \max \left[ 0, u'_{i,0} \left( \frac{\gamma_{i,0}\phi_{a_1}(0)}{p_{a_1}(0)} \right) \right]$ . This completes

the proof. ■

The solution of the original problem follows the same principles. Note that it suffices to solve the problem at any time  $t$ , and at any event  $a_t$ . The problem is

$$\begin{aligned} & \underset{\left\{ \begin{array}{l} z_i(a_s), t \geq s \\ a_s \subseteq a_t, a_s \in F_s \end{array} \right\}}{\text{Max}} && u_{i,t}(z_i(a_t)) + \sum_{s=t+1}^T \sum_{\substack{a_s \in F_s \\ a_s \subseteq a_t}} p_{a_s}(a_t) u_{i,s}(z_i(a_s)) \\ \text{s.t.} && z_i(a_t) + \sum_{s=t+1}^T \sum_{\substack{a_s \in F_s \\ a_s \subseteq a_t}} \phi_{a_s}(a_t) z_i(a_s) = c_i(a_t) + \sum_{s=t+1}^T \sum_{\substack{a_s \in F_s \\ a_s \not\subseteq a_t}} \phi_{a_s}(a_t) c_i(a_s) \\ && z_i(a_t) \geq 0, \quad z_i(a_s) \geq 0 \quad \forall a_s \in F_s, a_s \subseteq a_t \end{aligned}$$

The first order conditions (F.O.C.) for the above Lagrangian evaluated at  $c_i(a_t)$  and  $c_i(a_s)$  are;

$$\mathbf{c_i(a_t)}: \quad u'_{i,t}(c_i(a_t)) - \gamma_{i,a_t} = 0 \quad (7)$$

$$\mathbf{c_i(a_s)}: \quad p_{a_s}(a_t) u'_{i,s}(c_i(a_s)) - \gamma_{i,a_t} \phi_{a_s}(a_t) = 0 \quad (8)$$

$$\mathbf{K-T}: \quad \mu_{i,0} c_i(a_t) = 0 \quad (9)$$

$$\mu_{i,a_s} c_i(a_s) = 0 \quad (10)$$

$$\text{Thus, } \frac{p_{a_s}(a_t) u'_{i,s}(c_i(a_s))}{u'_{i,t}(c_i(a_t))} = \phi_{a_s}(a_t) \quad (11)$$

Letting  $\gamma_{i,a_t} = \gamma_i \frac{\phi_{a_t}}{p_{a_t}}$ , and using the definitions of  $p_{a_t}(a_t)$  and  $\phi_{a_t}(a_t)$ , the

optimal time- $t$  consumption can be written as

$$c_i(a_t) = \max\left[0, u'_{i,t}(\gamma_{i,a_t})\right] = \max\left[0, u'_{i,0}\left(\frac{\gamma_i \phi_{a_t}}{p_{a_t}}\right)\right] \quad (12)$$

From Equation (12), we can see that the optimal consumption for the  $i$ th individual depends on the prices of time-event contingent claim, and the associated probabilities of events in the economy at time  $t$ , and the Lagrange multiplier of the budget constraint. To derive the corresponding optimal sharing rules, we follow a methodology similar to Vanden.

The aggregate consumption in the economy at time  $t$ , and event  $a_t$  can be written as;

$$C(a_t) = \sum_{i=1}^I c_i(a_t) = \sum_{i=1}^I \max\left[0, u'_{i,t}\left(\frac{\gamma_i \phi_{a_t}}{p_{a_t}}\right)\right] \quad (13)$$

Now, define a real-valued function  $\Delta(x)$ , such that;

$$\Delta(x) = \sum_{i=1}^I \max\left[0, u'_{i,t}(\gamma_i x)\right] \quad (14)$$

$$\text{then } \Delta\left(\frac{\phi_{a_t}}{p_{a_t}}\right) = C(a_t), \text{ and } \frac{\phi_{a_t}}{p_{a_t}} = \Delta^{-1}(C(a_t)) \quad (15)$$

and the  $i$ th agent's optimal sharing rule becomes;

$$c_i(a_t) = \max[0, u_{i,t}'^{-1}(\gamma_i \Delta^{-1}(C(a_t)))] \quad (16)$$

In the above expression,  $\Delta^{-1}$  is the inverse mapping of the function  $\Delta$  on the interval where  $\Delta$  is strictly increasing. The closed form solution of  $\Delta^{-1}(C(a_t))$  for an economy with agents possessing quadratic utility with identical cautiousness is given below.

**Proposition 2.** For the given constants  $A_1 = \Delta\left(\frac{1}{\gamma_{2,t}}\right)$ ,  $A_2 = \Delta\left(\frac{1}{\gamma_{3,t}}\right)$ , ... ,

$A_{t-1} = \Delta\left(\frac{1}{\gamma_{1,t}}\right)$ , and the relation  $\gamma_{1,t}^{-1} > \gamma_{2,t}^{-1} > \dots > \gamma_{t-1,t}^{-1}$ , the closed form solution for

$\Delta^{-1}(C(a_t))$  is given as

$$\Delta^{-1}(C(a_t)) = \begin{cases} \frac{1 - bC(a_t)}{\gamma_{1,t}} & 0 < C(t) \leq A_1 \\ \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} & A_1 < C(t) \leq A_2 \\ \vdots & \\ \frac{I - bC(a_t)}{\bar{\gamma}_{1,t}} & A_{t-1} < C(t) \end{cases}$$

where  $\bar{\gamma}_{k,t} = \sum_{j=1}^k \gamma_{j,t}$ .

and the optimal sharing rule for the representative agent is given by

$$c_1(a_t) = C(a_t) - \sum_{j=1}^{I-1} \frac{\gamma_{1,t} \gamma_{j+1,t}}{\bar{\gamma}_{j,t} \bar{\gamma}_{j+1,t}} \max[0, C(a_t) - A_j]$$

$$\text{where } A_j = \frac{1}{b} \left( j - \frac{\bar{\gamma}_{j,t}}{\gamma_{j+1,t}} \right), \quad j = 1, 2, \dots, I-1 .$$

**Proof.** See the Appendix for the proof.

In general, we observe the following. A switch to a higher consumption interval (i.e. an increase in aggregate consumption from the  $i-1$ st bracket to the  $i$ th bracket) results in one more agent joining the economy. This last agent's optimal consumption is given by  $c_i(a_t) = \frac{\gamma_{i,t}}{\bar{\gamma}_{i,t}} (C(a_t) - A_i)$ . Furthermore, the remaining agents' optimal sharing rules are adjusted in such a way to compensate the inclusion of this last agent to the economy. That is, they will consume in a similar pattern when this agent was not present (i.e. as in the  $i-1$ st bracket) minus they will make an adjustment to compensate for the joining agent's consumption,  $c_i(a_t)$ , that is proportional to their shadow prices,  $\gamma_{j,t}$ 's, where  $j = 1, \dots, i-1$ .

Proposition 2 implies that the representative agent holds the aggregate consumption and  $I-1$  call options written on the aggregate consumption with strike

$$\text{prices, } A_j = \frac{1}{b} \left( j - \frac{\bar{\gamma}_{j,t}}{\gamma_{j+1,t}} \right), \quad j = 1, 2, \dots, I-1 .$$

The fact that the representative agent holds the optimal portfolio as in Proposition (2) in markets reconvening at each period has important pricing consequences. The setting of a multiperiod securities market with rational expectations equilibrium results in a C-CAPM, whereas the nonnegative wealth

constraints in our multiperiod setting results in a multibeta C-CAPM. The first beta is the covariance of the return of a risky asset with an asset that is highly correlated with the aggregate consumption, and the remaining  $I - 1$  betas are the covariances of the return of the asset with the returns of options with strike prices given in (40). This can be formalized with the following proposition.

**Proposition 3.** *The optimal portfolio in held by the representative agent indexed with  $i = 1$ , and having a quadratic utility, results in a multifactor conditional C-CAPM, where the first factor is the change in log aggregate consumption and the remaining  $I - 1$  factors are option returns with strike prices given by ,  $A_j = \frac{1}{b} \left( j - \frac{\bar{\gamma}_{j,t}}{\gamma_{j+1,t}} \right)$ ,  $j = 1, 2, \dots, I-1$ . The multifactor conditional C-CAPM is represented by*

$$E[\tilde{R}(t)|F_{t-1}] - R_f(t)\mathbf{1} = \beta_{Nc_1} \beta_{c_1c_1}^{-1} \{E[\tilde{R}_{c_1}(t)|F_{t-1}] - R_f(t)\mathbf{1}\}$$

where  $\beta_{Nc_1}$  is the  $N \times I$  variance-covariance matrix of the risky assets with the representative agent's optimal portfolio, and  $\beta_{c_1c_1}^{-1}$  is the  $I \times I$  variance matrix of the representative agent's optimal portfolio.

**Proof.** Let the sequence  $\{F_t, t = 0, 1, \dots, T\}$  be an information structure, such that the possible realizations of  $F_t$  from time 0 to time  $t$  generate a state space  $\Omega$ . Assume that the representative agent is endowed with this information structure and has a quadratic utility given by  $u_{1t}(c_1(t)) = c_1(t) - \frac{b}{2}c_1^2(t)$ . The utility function of the

representative agent is strictly concave and differentiable. Also assume that  $F_0$  is just  $\{\Omega\}$ .

The price of a long-lived security in this economy is given by

$$S_j(a_t, t) = \sum_{s=t+1}^T \sum_{\substack{a_s \in F_s \\ \{a_s \subseteq a_t\}}} \phi_{a_s}(a_t) X_j(a_s) \quad (17)$$

where  $X_j(a_s)$  is the dividend paid by security  $j$  in event  $a_s$ .<sup>5</sup> By using

Equation (11), we can rewrite the price of a long-lived security as

$$S_j(a_t, t) = \sum_{s=t+1}^T \sum_{\substack{a_s \in F_s \\ \{a_s \subseteq a_t\}}} \frac{p_{a_s}(a_t) u'_{1,s}(c_1(a_s))}{u'_{1,t}(c_1(a_t))} X_j(a_s) \quad (18)$$

By using the definition of  $S_j(a_{t-1}, t-1)$  from Equation (17), one can write

$$S_j(a_{t-1}, t-1) = \sum_{\substack{a_t \in F_t \\ \{a_t \subseteq a_{t-1}\}}} \frac{p_{a_t}(a_{t-1}) u'_{1,t}(c_1(a_t))}{u'_{1,t-1}(c_1(a_{t-1}))} [X_j(a_t) + S_j(a_t, t)] \quad (19)$$

Also by using the definitions of expected value, the random ex-dividend price of a long-lived security is given as

$$S_j(t-1) = E \left[ \sum_{s=t}^T \frac{u'_{1,s}(c_1(s))}{u'_{1,t-1}(c_1(t-1))} X_j(s) \middle| F_{t-1} \right] \quad (20)$$

$$S_j(t-1) = E \left[ \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} [X_j(t) + S_j(t)] \middle| F_{t-1} \right] \quad (21)$$

$$1 = E \left[ \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} [1 + \tilde{R}_j(t)] \middle| F_{t-1} \right] \quad (22)$$

is the expected return process for a long-lived security  $j$ .

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<sup>5</sup> A long-lived security is a complex security that is available for trading at all periods, and is composed of time-0 consumption good and a bundle of time-event contingent claims, and is represented by  $X = \{X_0, X_{a_t}; a_t \in F_t, t = 1, \dots, T\}$ , where  $X_0$  and  $X_{a_t}$  are the dividends paid at time 0 and at time  $t$  in event  $a_t$ , respectively, in units of consumption good.

From the definition of the covariance, equation (22) can be rewritten as;

$$1 = Cov \left[ \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))}, 1 + \tilde{R}_j(t) \middle| F_{t-1} \right] + E \left[ \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] E[1 + \tilde{R}_j(t) | F_{t-1}] \quad (23)$$

From Equation (22), the existence of a risk-free asset implies that

$$\frac{1}{1 + R_f(t)} = E \left[ \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] \quad (24)$$

Substituting (24) into (23), we have;

$$E[\tilde{R}_j(t) | F_{t-1}] - R_f(t) = -(1 + R_f(t)) Cov \left[ \tilde{R}_j(t), \frac{u'_{1,t}(c_1(t))}{u'_{1,t-1}(c_1(t-1))} \middle| F_{t-1} \right] \quad (25)$$

$$E[\tilde{R}_j(t) | F_{t-1}] - R_f(t) = -(1 + R_f(t)) Cov \left[ \tilde{R}_j(t), \frac{bc_1(t)}{bc_1(t-1)} \middle| F_{t-1} \right] \quad (26)$$

$$E[\tilde{R}_j(t) | F_{t-1}] - R_f(t) = -(1 + R_f(t)) Cov[\tilde{R}_j(t), \tilde{R}_{c_1}(t) | F_{t-1}] \quad (27)$$

Since this equation holds for any traded asset, or a portfolio of traded assets, it should also hold for the representative agent's portfolio,  $\tilde{R}_{c_1}(t) = (\tilde{R}_c(t), \tilde{R}_{o_2}(t), \dots, \tilde{R}_{o_{t-1}}(t))^T$ , where  $\tilde{R}_c(t)$  is the change in aggregate consumption, and  $\tilde{R}_{o_j}(t)$  is the return of the  $j^{th}$  option at time  $t$ . Thus,

$$E[\tilde{R}_{c_1}(t) | F_{t-1}] - R_f(t) = -(1 + R_f(t)) Cov[\tilde{R}_{c_1}^T(t), \tilde{R}_{c_1}(t) | F_{t-1}] \quad (28)$$

Substituting (28) into (27) gives

$$E[\tilde{R}_j(t) | F_{t-1}] - R_f(t) = \frac{Cov(\tilde{R}_j(t), \tilde{R}_{c_1}(t) | F_{t-1})}{Cov_t(\tilde{R}_{c_1}^T(t), \tilde{R}_{c_1}(t) | F_{t-1})} (E[\tilde{R}_{c_1}(t) | F_{t-1}] - R_f(t)) \quad (29)$$

In general for a vector of  $N$  risky assets,  $\tilde{R}(t) = (\tilde{R}_1(t), \tilde{R}_2(t), \dots, \tilde{R}_N(t))^T$ ;

$$E[\tilde{R}(t)|F_{t-1}] - R_f(t)\mathbf{1} = \frac{Cov(\tilde{R}(t), \tilde{R}_{c_1}(t)|F_{t-1})}{Cov(\tilde{R}_{c_1}^T(t), \tilde{R}_{c_1}(t)|F_{t-1})} (E[\tilde{R}_{c_1}(t)|F_{t-1}] - R_f(t)\mathbf{1}) \quad (30)$$

where  $Cov(\tilde{R}(t), \tilde{R}_{c_1}(t)|F_{t-1})Cov(\tilde{R}_{c_1}(t), \tilde{R}_{c_1}(t)|F_{t-1})^{-1}$  is an  $N \times I$  matrix of conditional betas for  $N$  risky assets with the return's of the representative agent's portfolio, and  $\tilde{R}_{c_1}(t) = (\tilde{R}_C(t), \tilde{R}_{O_2}(t), \dots, \tilde{R}_{O_{I-1}}(t))^T$  is the  $I \times 1$  vector of returns for the representative agent's portfolio.

Equation (30) can be written in a multibeta representation as,

$$E[\tilde{R}(t)|F_{t-1}] - R_f(t)\mathbf{1} = \beta_{N_{c_1}} \beta_{c_1 c_1}^{-1} \{E[\tilde{R}_{c_1}(t)|F_{t-1}] - R_f(t)\mathbf{1}\} \quad (31)$$

where

$$\beta_{N_{c_1}} = \begin{bmatrix} Cov_{t-1}(\tilde{R}_1(t), \tilde{R}_C(t)) & Cov_{t-1}(\tilde{R}_1(t), \tilde{R}_{O_1}(t)) & \cdots & Cov_{t-1}(\tilde{R}_1(t), \tilde{R}_{O_{I-1}}(t)) \\ Cov_{t-1}(\tilde{R}_2(t), \tilde{R}_C(t)) & Cov_{t-1}(\tilde{R}_2(t), \tilde{R}_{O_1}(t)) & \cdots & Cov_{t-1}(\tilde{R}_2(t), \tilde{R}_{O_{I-1}}(t)) \\ \vdots & \vdots & \ddots & \vdots \\ Cov_{t-1}(\tilde{R}_N(t), \tilde{R}_C(t)) & Cov_{t-1}(\tilde{R}_N(t), \tilde{R}_{O_1}(t)) & \cdots & Cov_{t-1}(\tilde{R}_N(t), \tilde{R}_{O_{I-1}}(t)) \end{bmatrix}$$

and

$$\beta_{c_1 c_1} = \begin{bmatrix} Var_{t-1}(\tilde{R}_C(t)) & Cov_{t-1}(\tilde{R}_C(t), \tilde{R}_{O_1}(t)) & \cdots & Cov_{t-1}(\tilde{R}_C(t), \tilde{R}_{O_{I-1}}(t)) \\ Cov_{t-1}(\tilde{R}_{O_1}(t), \tilde{R}_C(t)) & Var_{t-1}(\tilde{R}_{O_1}(t)) & \cdots & Cov_{t-1}(\tilde{R}_{O_1}(t), \tilde{R}_{O_{I-1}}(t)) \\ \vdots & \vdots & \ddots & \vdots \\ Cov_{t-1}(\tilde{R}_{O_{I-1}}(t), \tilde{R}_C(t)) & Cov_{t-1}(\tilde{R}_{O_{I-1}}(t), \tilde{R}_{O_1}(t)) & \cdots & Var_{t-1}(\tilde{R}_{O_{I-1}}(t)) \end{bmatrix}$$

This completes the proof. ■

Equation (31) is the main testable result of the outlined theory. It suggests that a multifactor conditional C-CAPM model with option returns as the factors, should explain the cross-sectional variation in securities returns. The following sections outline the econometric framework for the tests of (31), and present the empirical findings associated with these tests.

## 2 ECONOMETRIC SPECIFICATIONS

According to Equation (31) an asset's return at time  $t$ , conditional on the information at  $t-1$ , should be explained by the changes in the aggregate consumption and returns of options written on the aggregate consumption. To test this, we first specify the general versions of conditional (and unconditional) C-CAPM models used for testing the theory, and then present the data used to test them in this section. The following section presents the empirical results of these tests.

### 2.1 Conditional model

We start by the stochastic discount factor framework outlined by Harrison and Kreps (1979). Their existence theorem states that, in the absence of arbitrage, there exists a stochastic discount factor,  $m_{t+1}$ , which satisfies

$$E_t \left[ (1 + \tilde{R}_{j,t+1}) m_{t+1} \mid F_t \right] = 1 \quad (32)$$

where  $E_t$  denotes the mathematical expectation operator conditional on the information available at time  $t$ , and  $\tilde{R}_{j,t+1}$  is the net return of any traded asset  $j$ . The conditional form of the SDF is be represented by

$$m_{t+1} = a_t + b_t \tilde{R}_{e,t+1} \quad (33)$$

where  $\tilde{R}_{e,t+1}$  is the net return on an unobservable mean-variance efficient frontier.

The above conditional form implies a conditional beta representation given by

$$E_t[\tilde{R}_{j,t+1}] = R_{o,t} - b_t R_{o,t} \text{Var}_t(\tilde{R}_{e,t+1}) \beta_{j,t} \quad (34)$$

where  $R_{o,t}$  is the net return on a zero-beta portfolio that is uncorrelated with  $m_{t+1}$ ,

$$b_t = -\frac{E_t[\tilde{R}_{e,t+1}] - R_{o,t}}{R_{o,t} \text{Var}_t(\tilde{R}_{e,t+1})} \quad (35)$$

and

$$\beta_{j,t} = \frac{\text{Cov}_t(\tilde{R}_{j,t+1}, \tilde{R}_{e,t+1})}{\text{Var}_t(\tilde{R}_{e,t+1})} \quad (36)$$

The question here is how one can incorporate the information that investors use when they determine expected returns in Equation (32). Because the investors' true information set is unobservable, one has to find observable variables to proxy

for that information set. Cochrane (1996) shows that conditional asset pricing models can be tested via a conditioning time  $t$  information variable,  $z_t$ . One way of incorporating conditioning variable,  $z_t$ , into the model is to scale factor returns, as discussed in Cochrane (2001); and used in Cochrane (1996), Hodrick and Zhang (2001), and Lettau and Ludvigson (2001b). This is done by scaling the factors with  $z_t$ , thus modeling the parameters  $a_t$ , and  $b_t$  as linear functions of  $z_t$ , such that  $a_t = \gamma_0 + \gamma_1 z_t$ , and  $b_t = \eta_0 + \eta_1 z_t$ .

Plugging these equations into Equation (33), we have a scaled multifactor model with constant coefficients taking the form

$$\begin{aligned} m_{t+1} &= (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t) \tilde{R}_{e,t+1} \\ &= \gamma_0 + \gamma_1 z_t + \eta_0 \tilde{R}_{e,t+1} + \eta_1 z_t \tilde{R}_{e,t+1} \end{aligned} \quad (37)$$

The scaled multifactor model can be tested by rewriting the conditional factor model in Equation (32), as an unconditional factor model with constant coefficients  $\gamma_0, \gamma_1, \eta_0$ , and  $\eta_1$  as follows,

$$E\left[\left(1 + \tilde{R}_{j,t+1}\right) \left(\gamma_0 + \gamma_1 z_t + \eta_0 \tilde{R}_{e,t+1} + \eta_1 z_t \tilde{R}_{e,t+1}\right)\right] = 1 \quad (38)$$

In order to be able to test the theory's main predictions, we have to put more structure on the SDF,  $m_{t+1}$ , and on the unobservable mean-variance efficient frontier,  $\tilde{R}_{e,t+1}$ . Following Cochrane (1996), we consider a linear factor pricing model

with a vector of observable factors,  $f_t$ . For example, in the classical conditional CAPM tests,  $f_t$  is the return of the value-weighted market portfolio. The only requirement for the components of  $f_t$  is that the factors should be observable and relevant to the model.

Denote the vector  $F_{t+1} = (1, z_t, f_{t+1}^T, f_{t+1}^T z_t)^T$ , or in a more compact form  $F_{t+1} = (1, \bar{f}_{t+1}^T)^T$ , where  $\bar{f}_{t+1} = (z_t, f_{t+1}, f_{t+1} z_t)$ . The stochastic discount factor in equation (37) can be represented by

$$m_{t+1} = \delta^T F_{t+1} \quad (39)$$

where  $\delta = (\gamma_0, b^T)$  is a constant vector, and  $b = (\gamma_1, \eta_0^T, \eta_1^T)$  is the vector of constant coefficients on the variable factors,  $\bar{f}_{t+1}$ . Equation (39) implies an unconditional multifactor beta representation for asset  $j$ ,

$$E[\tilde{R}_{j,t+1}] = E[\tilde{R}_{0,t}] + \beta^T \lambda \quad (40)$$

where  $\beta = \frac{Cov(\tilde{R}_{j,t+1}, \bar{f})}{Cov(\bar{f}^T, \bar{f})}$  is a vector of regression coefficients from a multiple regression of returns on the variable factors.

## 2.2 Conditioning Variable

The choice of the conditioning variable is important because it summarizes the information that investors use while forming their expectations about securities returns. Due to its role in constructing the SDF one has to find a relevant and

theoretically sound variable. Regarding its success in explaining the cross-section of expected returns, we have decided to use the log consumption-wealth ratio that is advocated by Lettau and Ludvigson (2001a, 2001b). First of all it has a significant explanatory power in the conditional versions of C-CAPM, and on the theoretical side it summarizes agents' expectations of future returns on the market portfolio. Second, Cochrane (1996) shows that when the log consumption-wealth ratio is used as a conditioning variable, one can derive CAPM as special cases of C-CAPM. However, one problem with the log consumption-wealth ratio is that it is unobservable. In order to overcome this, we follow the methodology outlined by Lettau and Ludvigson (2001a, 2001b), and choose  $cay_t$  as an estimate of the log consumption-wealth ratio. Lettau and Ludvigson (2001a) argue that the log aggregate consumption,  $c_t$ , log asset wealth,  $a_t$ , and log labor earnings,  $y_t$ , are cointegrated, and they share a common trend. They define the trend term as  $cay_t$ , which is the cointegrating residual between  $c_t$ ,  $a_t$ , and  $y_t$ . Then,  $cay_t$  is defined to be  $cay_t = c_t - \omega a_t - (1 - \omega)y_t$ , where  $\omega$  denotes the share of nonhuman (asset) wealth,  $A_t$ , in total wealth,  $W_t$ .

The empirical work with consumption data has used expenditures on nondurables and services,  $c_{n,t}$ , as a measure of the aggregate consumption, and assumed that aggregate consumption is a constant multiple of nondurables and services consumption, i.e.  $c_t = \kappa c_{n,t}$ , where  $\kappa > 1$ . Thus,  $cay_t = c_{n,t} - \beta_a a_t - \beta_y y_t$ , where  $\beta_a = (1/\kappa)\omega$ , and  $\beta_y = (1/\kappa)(1 - \omega)$ .  $\beta_a$  and  $\beta_y$  are estimated using the following multivariate regression via OLS:

$$c_{n,t} = \alpha + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^k b_{a,i} \Delta a_{t-i} + \sum_{i=-k}^k b_{y,i} \Delta y_{t-i} + \varepsilon_i \quad (41)$$

where  $\Delta$  is the first difference operator. Then the estimated trend deviation is given by

$$c\hat{a}y_t \equiv c_{n,t} - \hat{\beta}_a a_t - \hat{\beta}_y y_t \quad (42)$$

where hats denote the estimated parameters.

### 2.3 The fundamental factors

Since consumption growth and option returns appear as factors in the asset pricing model of Equation (31), we assume that the SDF can be approximated as a linear function of consumption growth and returns on options written on the aggregate consumption. However, there do not exist any traded options written on the aggregate consumption, therefore we assume that there exists a function  $g(\cdot; F_t)$  such that  $\tilde{C}_t = g(\tilde{A}_t; F_t)$ , where  $\tilde{A}_t$  denotes the aggregate wealth at time  $t$ . We also assume that the S&P 500 index at time  $t$ , resembles a fairly well representation of, and is highly correlated with,  $\tilde{A}_t$ . Thus, instead of options written on the aggregate consumption, we use observable proxies, i.e. options written on the S&P 500 index to test our model.

More specifically, the vector of observable factors is chosen to be  $f_t = (\Delta c_t, \tilde{R}_o)$ , (or any subset of it), where  $\Delta c_t$  is the change in log consumption,

and  $\tilde{R}_o$  is an  $I - 1$  vector of option returns written on the S&P 500 index. Thus the general conditional (or unconditional) form of the stochastic factor would be

$$m_{t+1} = \gamma_0 + \gamma_1 cay_t + \eta_0^T f_{t+1} + \eta_1^T cay_t f_{t+1} \quad (43)$$

or subsets of it.

## 2.4 Data and Methodology

For all the econometric analyses, we use quarterly data. This is due to the announcement of the gross domestic product (GDP) data by U.S. Bureau and Economic Analysis (BEA) quarterly. The data covers the period 1990 Q1:2006Q1 for a total of 65 quarters (195 months). In estimating  $cay_t$ , the data for log consumption,  $c_t$ , log asset wealth,  $a_t$ , and log labor income,  $y_t$ , are downloaded from Martin Lettau's website.<sup>6</sup> The empirical tests use 10 portfolios sorted according to their market capitalizations, and 25 portfolios sorted according to size and book-to-market value ratios. Monthly return data for the portfolios and the risk-free rate are downloaded from Kenneth French's website.<sup>7</sup> The data for S&P 500 (SPX) call and put options are from the Chicago Board Options Exchange's (CBOE) Market Data Express (MDX). Finally, for the market portfolio, CRSP's value weighted index on all NYSE, AMEX and NASDAQ stocks are used.

For  $cay_t$  estimation, we have used a variety of leads and lags  $k = 1, \dots, 8$  and report here the results for  $k = 8$ . The results for other lags are similar. The estimated

<sup>6</sup> [http://pages.stern.nyu.edu/~mlettau/data\\_cay.html](http://pages.stern.nyu.edu/~mlettau/data_cay.html)

<sup>7</sup> [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

value for  $cay_t$  for the above test period is found to be  $\widehat{cay}_t \equiv c_t - 0.18a_t - 0.61y_t - 1.83$ . This estimated value is used as the conditioning variable in the empirical tests of conditional C-CAPM models.

The method for calculating daily option returns is as follows. First, we choose daily closing prices for SPX call and puts for a variety of strike prices and maturities and for the class of non-leap options. Second, options that expire during the following calendar month are identified. The reason for choosing options that expire the next calendar month is that they are the most liquid data among various maturities.<sup>8</sup> Then, options that expire within 14 days are excluded from the sample, because they show large deviations in trading volumes, which casts doubt on the reliability of their pricing associated with increased volatility.<sup>9</sup> Then, we group the call and put options according to their moneyness levels. More specifically, for the vector of observable factors,  $f_t = (\Delta c_t, \tilde{R}_O)$ , we choose 3 option classes according to their moneyness levels. Thus,  $O = (c_{at}, c_{in}, c_{out})$ , or  $O = (p_{at}, p_{in}, p_{out})$  where  $c_{at}$ ,  $c_{in}$ , and  $c_{out}$  stand for at-the-money, in-the-money, and out-of-the-money call options; and  $p_{at}$ ,  $p_{in}$ , and  $p_{out}$  stand for at-the-money, in-the-money, and out-of-the-money put options, respectively. We have used the following criteria for moneyness classification.

$c_{at}, p_{at}$	$c_{in}, p_{out}$	$c_{out}, p_{in}$
$-5 \leq S - K \leq 5$	$1.03 \leq S / K \leq 1.06$	$0.94 \leq S / K \leq 0.97$

<sup>8</sup> According to Buraschi and Jackwerth (2001), most of the trading activity in S&P500 options is concentrated in the nearest (0-30 days to expiry) and second nearest (30-60 days to expiry) contracts.

<sup>9</sup> Stoll and Whaley (1987) report abnormal trading volumes for options close to expiry.

Classifying at-the-money options with moneyness level (S-K) between -5 and +5 follows Coval and Shumway (2001), and is chosen in order to guarantee that there are at least two and at most three options around the spot price. The classification for in-the-money and out-of-the-money options follows Bakshi, Cao, and Chen (1997). Returns for options for the above six categories are then calculated. We use raw returns, because Coval and Shumway (2001) report that using log returns could be quite problematic. The daily average option return, is then, the equally weighted average of the returns of options that belong to that category. These returns are then cumulated to quarterly returns ending up with 65 quarterly (195 monthly return data for the six categories of options. Table 3.2 reports the summary statistics for daily call and put option returns for different moneyness levels.

**TABLE 3.2**  
**Summary Statistics for SPX options**

	$C_{at}$	$C_{in}$	$C_{out}$	$P_{at}$	$P_{in}$	$P_{out}$
Mean	0,35	0,17	0.84	-1.27	-0.71	-1.76
Median	-0,58	-0,09	-4.80	-3.25	-1.69	-5.99
Minimum	-74.73	-66.23	-84.52	-69.02	-55.16	-75.95
Maximum	131.40	78.53	299.05	221.85	130.12	407.55
Skewness	0.74	0.33	1.65	1.1591	0.61	2.31
Kurtosis	2.04	1.67	5.03	4.6268	2.55	15.84

*Note.* This table reports the summary statistics for the returns of daily call and put option returns on the S&P 500 index. The sample covers the period January 1990 to March 2006. The return figures are in percentages.

Daily returns for at-the-money call and put options are consistent with what has been documented in the literature [Coval and Shumway (2001), Vanden (2004)]. Daily average returns for call (put) options are positive (negative) regardless of their moneyness levels. Furthermore returns increase in absolute values as one goes from

in-the-money options out-of-the-money options. It is clear that out-of-the-money calls (puts) are the biggest earners (losers) among the three given moneyness levels.

The next section presents findings from the time-series, cross-sectional and GMM-SDF estimations for a variety of conditional and unconditional C-CAPM models.

### 3 EMPIRICAL RESULTS

In this section we test four versions of C-CAPM.

- i) The unconditional C-CAPM

$$f_t = (\Delta c_t)$$

- ii) Unconditional C-CAPM with call and put options

$$f_t = (\Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t})$$

$$f_t = (\Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t})$$

- iii) Conditional C-CAPM

$$f_t = (c\hat{a}y_t, \Delta c_t, c\hat{a}y_t \Delta c_{t+1})$$

- iv) Conditional C-CAPM with call and put options

$$f_t = (c\hat{a}y_t, \Delta c_t, R_{c_{at},t}, R_{c_{in},t}, R_{c_{out},t}, c\hat{a}y_t \Delta c_{t+1}, c\hat{a}y_t R_{c_{at},t+1}, c\hat{a}y_t R_{c_{in},t+1}, c\hat{a}y_t R_{c_{out},t+1})$$

$$f_t = (c\hat{a}y_t, \Delta c_t, R_{p_{at},t}, R_{p_{in},t}, R_{p_{out},t}, c\hat{a}y_t \Delta c_{t+1}, c\hat{a}y_t R_{p_{at},t+1}, c\hat{a}y_t R_{p_{in},t+1}, c\hat{a}y_t R_{p_{out},t+1})$$

### 3.1 Time Series Regressions

First, we test whether option returns explain the factor loadings of different portfolios. To do this, we take the unconditional C-CAPM with options (ii) as our base model. Thus, the empirical model to be tested is

$$R_{it} = \alpha_i + \beta_i f_t^T + \varepsilon_{it} \quad (44)$$

where  $R_{it}$ 's are realized quarterly excess returns of 10 size, and 25 size and book-to-market (BV/MV) portfolios,  $\beta_i$  is a row vector of betas for the  $i$ th portfolio, and  $f_t$  is as given in ii. Tables 3.3 and 3.4 report the time series regression results.

As can be seen from Tables 3.3 and 3.4, option returns help explain the variation in returns of the chosen portfolios. Panel A of Table 3.3 report the regression results of excess portfolio returns formed according to size on the aggregate consumption and excess call returns. It is seen that change in aggregate consumption, together with the excess returns of call options help explain the variation in the returns of size portfolios. The adjusted  $R^2$ 's range from 0.20 (for smallest size portfolio) to 0.58 (for biggest size portfolio), and the model tends to explain the returns of bigger size portfolios better. Furthermore, although the intercepts terms are individually significant, a low GRS F-statistic with a p-value of 0.763 rejects the joint test of significance of intercept terms, i.e they are not jointly significantly different from zero.

**TABLE 3.3**  
**10 size regressions**

<i>PANEL A: 10 size regressions using excess call returns</i>											
	$\alpha_i$	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,c_{at}}$	t-stat	$\beta_{i,c_{in}}$	t-stat	$\beta_{i,c_{out}}$	t-stat	Adj. R <sup>2</sup>
<b>Small1</b>	0.0301	2.71 <sup>***</sup>	-0.0123	-1.60	-0.0152	-1.69 <sup>*</sup>	0.0877	2.55 <sup>**</sup>	-0.0180	-1.84 <sup>*</sup>	0.20
<b>Decile2</b>	0.0259	2.68 <sup>***</sup>	-0.0193	-2.54 <sup>**</sup>	-0.0196	-2.45 <sup>**</sup>	0.0988	3.03 <sup>***</sup>	-0.0182	-1.84 <sup>*</sup>	0.31
<b>Decile3</b>	0.0236	3.00 <sup>***</sup>	-0.0190	-2.72 <sup>***</sup>	-0.0170	-2.22 <sup>**</sup>	0.0922	3.57 <sup>***</sup>	-0.0169	-2.08 <sup>**</sup>	0.35
<b>Decile4</b>	0.0194	2.76 <sup>***</sup>	-0.0167	-2.47 <sup>**</sup>	-0.0192	-2.80 <sup>***</sup>	0.0963	3.75 <sup>***</sup>	-0.0168	-2.18 <sup>**</sup>	0.38
<b>Decile5</b>	0.0219	3.05 <sup>***</sup>	-0.0210	-3.35 <sup>***</sup>	-0.0197	-3.05 <sup>***</sup>	0.0964	4.00 <sup>***</sup>	-0.0171	-2.44 <sup>**</sup>	0.43
<b>Decile6</b>	0.0188	2.93 <sup>***</sup>	-0.0184	-3.08 <sup>***</sup>	-0.0145	-2.70 <sup>***</sup>	0.0849	3.95 <sup>***</sup>	-0.0153	-2.33 <sup>**</sup>	0.45
<b>Decile7</b>	0.0240	3.61 <sup>***</sup>	-0.0189	-2.92 <sup>***</sup>	-0.0135	-2.04 <sup>**</sup>	0.0812	3.33 <sup>***</sup>	-0.0149	-1.83 <sup>*</sup>	0.44
<b>Decile8</b>	0.0213	2.99 <sup>***</sup>	-0.0178	-3.18 <sup>***</sup>	-0.0142	-2.48 <sup>**</sup>	0.0784	3.26 <sup>***</sup>	-0.0135	-1.82 <sup>*</sup>	0.41
<b>Decile9</b>	0.0209	3.90 <sup>***</sup>	-0.0155	-3.10 <sup>***</sup>	-0.0083	-1.88 <sup>*</sup>	0.0653	3.40 <sup>***</sup>	-0.0114	-1.87 <sup>*</sup>	0.46
<b>Big10</b>	0.0138	2.04 <sup>**</sup>	-0.0147	-3.46 <sup>***</sup>	-0.0055	-1.30	0.0646	3.95 <sup>***</sup>	-0.0107	-1.83 <sup>*</sup>	0.58
GRS (10,51) = 0.6517 (0.763)											
<i>PANEL B: 10 size regressions using excess put returns</i>											
	$\alpha_i$	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,p_{at}}$	t-stat	$\beta_{i,p_{in}}$	t-stat	$\beta_{i,p_{out}}$	t-stat	Adj. R <sup>2</sup>
<b>Small1</b>	0.0044	0.22	0.0033	0.56	-0.0246	-1.11	-0.0601	-1.91 <sup>*</sup>	0.0141	0.45	0.16
<b>Decile2</b>	0.0049	0.24	-0.0042	-0.45	-0.0205	-0.98	-0.0747	-2.80 <sup>***</sup>	0.0245	0.89	0.25
<b>Decile3</b>	-0.0045	-0.02	-0.0045	-0.52	-0.0185	-0.97	-0.0715	-2.67 <sup>***</sup>	0.0176	0.66	0.30
<b>Decile4</b>	-0.0036	-0.21	-0.0016	-0.19	-0.0202	-1.18	-0.0755	-2.93 <sup>***</sup>	0.0224	0.89	0.33
<b>Decile5</b>	-0.0034	-0.23	-0.0053	-0.69	-0.0205	-1.73 <sup>*</sup>	-0.0743	-3.21 <sup>***</sup>	0.0191	0.87	0.39
<b>Decile6</b>	-0.0065	-0.51	-0.0026	-0.39	-0.0256	-2.48 <sup>**</sup>	-0.0698	-3.58 <sup>***</sup>	0.0210	1.11	0.44
<b>Decile7</b>	-0.0010	-0.08	-0.0047	-0.64	-0.0227	-2.24 <sup>**</sup>	-0.0623	-3.92 <sup>***</sup>	0.0152	0.92	0.41
<b>Decile8</b>	-0.0010	-0.07	-0.0038	-0.56	-0.0188	-1.53	-0.0706	-3.53 <sup>***</sup>	0.0204	1.02	0.42
<b>Decile9</b>	-0.0005	-0.04	-0.0035	-0.49	-0.0170	-1.48	-0.0644	-3.90 <sup>***</sup>	0.0171	1.00	0.49
<b>Big10</b>	-0.0067	-0.39	-0.0050	-0.66	-0.0125	-0.76	-0.0653	-5.19 <sup>***</sup>	0.0150	1.03	0.53
GRS (10,51) = 0.4451 (0.917)											

*Note.* This table reports quarterly time-series regression results of excess returns of CRSP's size deciles on change in log consumption and excess call and put returns. i.e.  $f_t = (\Delta c_t, R_{c_{at,t}}, R_{c_{in,t}}, R_{c_{out,t}})$ , and  $f_t = (\Delta c_t, R_{p_{at,t}}, R_{p_{in,t}}, R_{p_{out,t}})$  for Panels A, and B, respectively. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

Looking at Panel B of Table 3.3, it can be seen that in-the-money-put returns are significant in among all size portfolios. The adjusted  $R^2$ 's are slightly lower than the model with excess call returns, and range from 0.16 (smallest size portfolio) to 0.53 (biggest size portfolio), and the tendency to explain the returns of bigger size portfolios remain. The GRS F-statistic is slightly lower with a p-value of 0.917, and clearly rejects the joint test of significance of intercept terms.

Next we test whether option returns help explain the returns of 25 portfolios formed according to size and book-to-market. Panel A of Table 3.4 reports the results for the model with excess call returns. We see that although the significance of the beta coefficients for out-of-the-money calls drop slightly, the results are quite consistent with the previous findings. The factor loadings for at-the-money call returns are significant for 22 of 25 portfolios, and moreover in-the-money call returns are significant across all portfolios. The adjusted  $R^2$ 's range from 0.20 to 0.51, and a high GRS F-test statistic with a p-value of 0.162 cannot reject the joint significance of the intercept terms. The results in Panel B of Table 3.4 are somewhat similar with that of Table 3. The returns of in-the-money puts are significant in 15 portfolios, and the model with put options again fares well in the biggest size quintile in terms of explanatory power, with adjusted  $R^2$ 's ranging from 0.13 to 0.48. The joint significance of the intercept terms cannot be rejected due to the high GRS F-statistic with a p-value of 0.001.

**TABLE 3.4**  
**25 size and book-to-market regressions**

*PANEL A: 25 size and book-to-market regressions using excess call returns*

	$\alpha_i$	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,c_{at}}$	t-stat	$\beta_{i,c_{in}}$	t-stat	$\beta_{i,c_{out}}$	t-stat	Adj. R <sup>2</sup>
<b>SL</b>	0.0027	0.17	-0.0304	-3.47***	-0.0232	-1.94*	0.1307	2.59**	-0.0295	-2.01**	0.29
<b>S2</b>	0.0288	2.52**	-0.0208	-2.76***	-0.0189	-1.97*	0.1036	2.81***	-0.0218	-1.95*	0.27
<b>S3</b>	0.0321	3.53***	-0.0123	-1.94*	-0.0183	-2.62**	0.0824	3.28***	-0.0137	-1.70*	0.24
<b>S4</b>	0.0397	4.14***	-0.0129	-1.77*	-0.0141	-2.38**	0.0691	3.19***	-0.0120	-1.69*	0.20
<b>SH</b>	0.0401	3.99***	-0.0089	-1.32	-0.0180	-1.87*	0.0854	2.72***	-0.0133	-1.40	0.21
<b>2L</b>	0.0114	1.08	-0.0274	-3.34***	-0.0181	-1.98*	0.1131	3.15***	-0.0231	-2.21**	0.36
<b>22</b>	0.0182	2.32**	-0.0166	-2.34**	-0.0176	-2.58**	0.0902	3.78***	-0.0156	-2.29**	0.33
<b>23</b>	0.0298	3.60***	-0.0144	-2.23**	-0.0180	-2.40**	0.0787	3.70***	-0.0135	-1.72*	0.30
<b>24</b>	0.0312	3.62***	-0.0105	-1.28	-0.0161	-2.41**	0.0735	3.71***	-0.0109	-1.68*	0.25
<b>2H</b>	0.0290	3.01***	-0.0077	-1.35	-0.0198	-2.15**	0.0880	3.13***	-0.0131	-1.54	0.25
<b>3L</b>	0.0125	1.26	-0.0306	-3.18***	-0.0170	-2.09**	0.1105	3.44***	-0.0236	-2.65**	0.42
<b>32</b>	0.0230	3.16***	-0.0180	-2.65**	-0.0194	-3.91***	0.0825	4.47***	-0.0112	-2.19**	0.39
<b>33</b>	0.0249	3.20***	-0.0126	-2.26**	-0.0117	-2.07**	0.0634	3.88***	-0.0098	-1.50	0.31
<b>34</b>	0.0234	2.51**	-0.0089	-1.46	-0.0147	-2.49**	0.0721	3.33***	-0.0115	-1.69*	0.27
<b>3H</b>	0.0337	3.29***	-0.0080	-1.41	-0.0161	-2.45**	0.0753	3.13***	-0.0109	-1.51	0.23
<b>4L</b>	0.0214	1.94*	-0.0327	-3.41***	-0.0155	-1.90*	0.0939	2.89***	-0.0189	-2.04**	0.42
<b>42</b>	0.0226	3.53***	-0.0137	-2.46**	-0.0099	-2.02**	0.0630	3.41***	-0.0093	-1.53	0.35
<b>43</b>	0.0261	3.43***	-0.0099	-1.63	-0.0101	-1.88*	0.0649	3.35***	-0.0111	-1.57	0.29
<b>44</b>	0.0270	3.25***	-0.0080	-1.61	-0.0124	-2.54**	0.0665	3.35***	-0.0099	-1.53	0.30
<b>4H</b>	0.0235	3.06***	-0.0028	-0.70	-0.0112	-1.74*	0.0649	2.65**	-0.0093	-1.06	0.22
<b>5L</b>	0.0159	1.87*	-0.0193	-3.86***	-0.0035	-0.65	0.0642	3.46***	-0.0122	-2.07**	0.51
<b>52</b>	0.0196	3.94***	-0.0115	-2.50**	-0.0066	-1.80*	0.0637	4.44***	-0.0096	-1.82*	0.51
<b>53</b>	0.0195	3.10***	-0.0067	-1.15	-0.0079	-2.09**	0.0546	4.05***	-0.0062	-1.12	0.38
<b>54</b>	0.0173	2.18**	-0.0039	-0.77	-0.0038	-0.96	0.0514	3.41***	-0.0079	-1.32	0.31
<b>5H</b>	0.0166	1.59	-0.0078	-1.32	-0.0065	-1.14	0.0549	2.41**	-0.0086	-1.14	0.27

GRS (25,36) = 1.4273 (0.162)

PANEL B: 25 size and book-to-market regressions using excess put returns

	$\alpha_i$	t-stat	$\beta_{i,\Delta c}$	t-stat	$\beta_{i,p_{at}}$	t-stat	$\beta_{i,p_{in}}$	t-stat	$\beta_{i,p_{out}}$	t-stat	Adj. R <sup>2</sup>
<b>SL</b>	-0.0313	-1.27	-0.0079	-0.77	-0.0132	-0.55	-0.1126	-3.56***	0.0204	0.71	0.29
<b>S2</b>	-0.0014	-0.07	-0.0038	-0.48	-0.0105	-0.52	-0.0848	-2.81***	0.0097	0.32	0.27
<b>S3</b>	0.0121	0.68	0.0003	0.06	-0.0203	-1.06	-0.0560	-2.28**	0.0166	0.64	0.18
<b>S4</b>	0.0190	1.02	-0.0011	-0.16	-0.0242	-1.18	-0.0412	-1.42	0.0115	0.40	0.16
<b>SH</b>	0.0190	0.96	0.0033	0.55	-0.0248	-1.21	-0.0529	-1.49	0.0170	0.50	0.13
<b>2L</b>	-0.0152	-0.70	-0.0082	-0.79	-0.0128	-0.58	-0.1121	-4.09***	0.0303	1.19	0.39
<b>22</b>	-0.0041	-0.22	-0.0052	-0.54	-0.0114	-0.57	-0.0648	-2.16**	0.0108	0.37	0.25
<b>23</b>	0.0067	0.41	-0.0006	-0.09	-0.0245	-1.65	-0.0502	-1.68*	0.0130	0.49	0.26
<b>24</b>	0.0055	0.32	0.0025	0.30	-0.0302	-2.07**	-0.0412	-1.34	0.0098	0.34	0.20
<b>2H</b>	0.0013	0.07	0.0037	0.62	-0.0185	-1.10	-0.0492	-1.43	0.0015	0.05	0.15
<b>3L</b>	-0.0088	-0.45	-0.0111	-0.96	-0.0181	-0.91	-0.1093	-5.19***	0.0398	1.99*	0.43
<b>32</b>	-0.0025	-0.20	-0.0045	-0.52	-0.0236	-2.50**	-0.0590	-2.47**	0.0141	0.64	0.35
<b>33</b>	-0.0023	-0.19	-0.0005	-0.11	-0.0241	-2.75***	-0.0421	-1.60	0.0037	0.16	0.31
<b>34</b>	-0.0073	-0.58	0.0041	0.86	-0.0333	-5.36***	-0.0293	-0.96	-0.0008	-0.03	0.21
<b>3H</b>	0.0002	0.01	0.0019	0.53	-0.0127	-1.11	-0.0400	-1.12	-0.0149	-0.48	0.19
<b>4L</b>	0.0018	0.09	-0.0165	-1.44	-0.0154	-0.79	-0.0910	-4.52***	0.0309	1.48	0.43
<b>42</b>	0.0001	0.01	-0.0032	-0.52	-0.0214	-2.33**	-0.0471	-1.87*	0.0102	0.45	0.30
<b>43</b>	-0.0039	-0.33	0.0028	0.55	-0.0253	-3.51***	-0.0422	-1.47	0.0008	0.03	0.19
<b>44</b>	-0.0011	-0.08	0.0029	0.52	-0.0194	-2.56**	-0.0428	-1.66*	-0.0010	-0.04	0.27
<b>4H</b>	0.0032	0.20	0.0045	0.90	-0.0112	-1.00	-0.0429	-1.42	0.0025	0.10	0.13
<b>5L</b>	-0.0007	-0.03	-0.0099	-1.32	-0.0073	-0.36	-0.0735	-4.83***	0.0202	1.21	0.48
<b>52</b>	0.0010	0.08	-0.0038	-0.52	-0.0144	-1.54	-0.0515	-2.98***	0.0118	0.68	0.36
<b>53</b>	-0.0049	-0.46	0.0012	0.18	-0.0157	-1.47	-0.0430	-2.61**	0.0030	0.16	0.31
<b>54</b>	-0.0097	-0.72	0.0063	1.00	-0.0247	-2.32**	-0.0397	-1.82*	0.0046	0.24	0.28
<b>5H</b>	-0.0205	-1.65	-0.0065	-2.12	0.0056	0.56	-0.0342	-1.32	-0.0365	-1.75*	0.33

GRS (25,36) = 3.3280 (0.001)

Note. This table reports quarterly time-series regression results of excess returns of CRSP's size deciles on change in log consumption and excess call and put returns. i.e.  $f_t = (\Delta c_t, R_{c_{at,t}}, R_{c_{in,t}}, R_{c_{out,t}})$ , and  $f_t = (\Delta c_t, R_{p_{at,t}}, R_{p_{in,t}}, R_{p_{out,t}})$  for Panels A, and B, respectively. \*\*\*, \*\*, \* denote 0.01, 0.05, and 0.10 significance levels, respectively. All t-values are corrected for autocorrelation (with lag=3) and heteroskedasticity as suggested by Newey and West (1987). GRS F-Test reported at the bottom of the table is from Gibbons, Ross, and Shanken (1989).

Overall the above results indicate that the call and put returns explain a significant amount of variation in securities returns. Thus, the results favor the hypothesis that option returns are useful tools in explaining securities returns. In the following two subsections, we formalize these findings by testing whether the risk premiums for option returns are priced or not.

### 3.2 Fama-MacBeth Estimations

In order to have a formal comparison between the proposed 4 models, and examine the power of various beta representations to explain the cross section of expected returns, we perform Fama-MacBeth regressions and estimate the associated risk premia for each model. The model to be tested is

$$E[\tilde{R}_{j,t+1}] = \alpha_j + \beta_j^T \lambda \quad (45)$$

where  $\beta_j = \frac{\text{Cov}(\tilde{R}_{j,t+1}, f_t)}{\text{Cov}(f_t^T, f_t)}$  is a vector of regression coefficients from

a multiple regression of quarterly returns on the variable factors, given in i, ii, iii, iv, and  $R_{j,t+1}$  are the quarterly returns of 25 portfolios sorted according to size and book-to-market.

The procedure to estimate  $\lambda$  is as follows. In the first pass, portfolio betas are estimated from a single multiple time-series regression via Equation (45). Due to having a data set for 65 quarters, instead of using the 5-year rolling-window approach, we use a full sample period. In the second pass, a cross-sectional regression is run at each time period, with full-sample betas obtained from the first

pass regressions. We estimate the intercept term and risk premia,  $\alpha_i$  and  $\lambda_j$ 's, as the average of these cross-sectional regression estimates, as outlined by Fama and MacBeth (1973).

Table 3.5 gives the results of Fama-MacBeth estimations. As can be seen from Row 1, the unconditional C-CAPM is very poor in explaining the cross section of expected securities returns. The results are consistent with the existing C-CAPM literature. The risk premium for consumption is insignificant, and the adjusted  $R^2$  indicates that only 14 percent of cross sectional variation of securities returns is explained by the unconditional C-CAPM. Row 2 confirms the findings of Lettau and Ludvigson (2001b) that the conditional C-CAPM (using  $c\hat{a}y_t$  as the conditioning variable) performs superior to its unconditional counterpart. The scaled factor is significant and the model explains 34 percent of the cross-sectional variation in securities returns.

One important point to be noted here is that the estimated coefficients of scaled variables, i.e.  $\lambda_{1,\Delta c}$ , should not be taken as the classical risk premia in unconditional models. As discussed by Lettau and Ludvigson (2001b) we should take into account the fact that each scaled unconditional multifactor model has an associated conditional model from which it is derived. Thus, the true risk prices for period  $t$  should actually be  $\lambda_t$ . However, the scaled multifactor model uses the unconditional covariance matrix  $Cov(f_{t+1}, f_{t+1}^T)$ , instead of the conditional covariance matrix  $Cov_t(f_{t+1}, f_{t+1}^T)$ , which is needed to estimate the true period  $t$  risk premia. Thus there is no simple relationship between the actual period  $t$  risk premia,  $\lambda_t$ , and

the estimated scaled  $\lambda$ 's, therefore the scaled coefficient estimates should not be taken as the true risk prices.

Row 3 presents the results for the unconditional version of C-CAPM when call returns are included. Looking at the Shanken corrected t-statistics, it is seen that the returns of at-the-money-call options have a negative and significant risk premium. Furthermore, when call option returns are included there is a significant improvement in the explanatory power of the unconditional model, from 14 percent to 47 percent. Similarly, Row 4 presents the results of unconditional C-CAPM with put returns included. The risk premium for out-of-money put returns is significant and positive. Although the adjusted  $R^2$  is lower than the model using call returns, it is nevertheless higher than the unconditional C-CAPM. The two results confirm that the inclusion of option returns help increase the explanatory power of C-CAPM.

Finally, Rows 5 and 6 present the results for the model that is predicted by the theory outlined here. The conditional version of C-CAPM, which uses  $c\hat{a}y_t$  as the conditioning variable, and option returns as fundamental factors perform significantly better than all the three previously tested models. Looking at Row 5, we can see that the previous significant risk premium of at-the-money call returns, and the significant coefficient of scaled consumption remain, and in addition, the scaled at-the-money calls also have a significant coefficient at 10 percent level. Furthermore, the overall explanatory power of the model rises to 61 percent. Similarly, Row 6 preserves the significant coefficients of the previous out-of-money put returns, and scaled consumption. Although none of the scaled put returns are significant, the overall explanatory power of the model is 53 percent, which is higher than its unconditional counterpart in Row 4.

**TABLE 3.5**  
**Fama-MacBeth Regressions**

	<u>Fundamental Factors</u>									<u>Scaled Factors</u>							
	$\alpha_i$	$\lambda_{cay}$	$\lambda_{o,\Delta c}$	$\lambda_{o,c_{at}}$	$\lambda_{o,c_{in}}$	$\lambda_{o,c_{out}}$	$\lambda_{o,p_{at}}$	$\lambda_{o,p_{in}}$	$\lambda_{o,p_{out}}$	$\lambda_{1,\Delta c}$	$\lambda_{1,c_{at}}$	$\lambda_{1,c_{in}}$	$\lambda_{1,c_{out}}$	$\lambda_{1,p_{at}}$	$\lambda_{1,p_{in}}$	$\lambda_{1,p_{out}}$	Adj. R <sup>2</sup>
<b>Row1</b>	2.63 (2.45 <sup>**</sup> ) (2.41 <sup>**</sup> )		0.17 (0.75) (0.74)														0.14
<b>Row2</b>	3.73 (2.63 <sup>**</sup> ) (2.46 <sup>**</sup> )	-0.11 (-0.23) (-0.22)	0.03 (0.19) (0.18)							0.04 (2.19 <sup>**</sup> ) (2.07 <sup>**</sup> )							0.34
<b>Row3</b>	5.05 (4.13 <sup>***</sup> ) (3.60 <sup>***</sup> )		0.16 (0.40) (0.35)	-2.22 (-2.64 <sup>**</sup> ) (-2.32 <sup>**</sup> )	-0.76 (-1.83 <sup>*</sup> ) (-1.59)	-1.42 (-1.19) (-1.04)											0.47
<b>Row4</b>	3.60 (3.32 <sup>***</sup> ) (2.87 <sup>***</sup> )		0.26 (0.66) (0.57)				-0.07 (-0.24) (-0.21)	0.32 (1.41) (1.22)	0.44 (2.40 <sup>**</sup> ) (2.09 <sup>**</sup> )								0.37
<b>Row5</b>	4.85 (3.81 <sup>***</sup> ) (3.16 <sup>***</sup> )	-0.05 (-1.04) (-0.67)	0.03 (0.20) (0.16)	-1.81 (-2.69 <sup>**</sup> ) (-2.10 <sup>**</sup> )	-0.84 (-2.01 <sup>**</sup> ) (-1.57)	-0.38 (-0.38) (-0.30)				0.01 (2.32 <sup>**</sup> ) (1.81 <sup>*</sup> )	0.04 (2.29 <sup>**</sup> ) (1.78 <sup>*</sup> )	0.02 (1.98 <sup>*</sup> ) (1.54)	0.01 (0.37) (0.29)				0.61
<b>Row6</b>	3.84 (2.71 <sup>***</sup> ) (2.31 <sup>**</sup> )	-0.06 (-1.55) (-1.17)	-0.04 (-0.26) (-0.18)				0.23 (0.97) (0.80)	0.29 (1.56) (1.28)	0.48 (2.63 <sup>**</sup> ) (2.15 <sup>**</sup> )	0.02 (2.78 <sup>***</sup> ) (2.27 <sup>**</sup> )				0.02 (0.24) (0.20)	-0.04 (-0.84) (-0.68)	-0.10 (-1.89 <sup>*</sup> ) (-1.54)	0.53

Note. This table gives the estimates for the cross-sectional Fama-MacBeth regression model  $E[\tilde{R}_{j,t+1}] = \alpha_i + \beta_j^T \lambda$  where  $\beta_j$ 's are estimated by a single time-series regression via Equation 62 using a full sample period, and using  $f_t$  given by i, ii, iii, iv. The estimated coefficients from the second-pass are  $\lambda = (\lambda_{cay}, \lambda_o, \lambda_1)$ , where  $\lambda_o$ , and  $\lambda_1$  denote the vector of coefficients for fundamental factors and scaled factors.  $\lambda_o = (\lambda_{o,\Delta c})$  for Rows 1 and 2,  $\lambda_o = (\lambda_{o,\Delta c}, \lambda_{o,c_{at}}, \lambda_{o,c_{in}}, \lambda_{o,c_{out}})$  for Rows 3 and 5, and  $\lambda_o = (\lambda_{o,\Delta c}, \lambda_{o,p_{at}}, \lambda_{o,p_{in}}, \lambda_{o,p_{out}})$  for Rows 4 and 6. Subsequently,  $\lambda_1 = (\lambda_{1,\Delta c})$  for Row 2,  $\lambda_1 = (\lambda_{1,\Delta c}, \lambda_{1,c_{at}}, \lambda_{1,c_{in}}, \lambda_{1,c_{out}})$  for Row 5, and  $\lambda_1 = (\lambda_{1,\Delta c}, \lambda_{1,p_{at}}, \lambda_{1,p_{in}}, \lambda_{1,p_{out}})$  for Row 6. The term  $R_{j,t+1}$  is the return on 25 Fama-French portfolios (j=1,2,...,25) in quarter t (1990Q1:2006Q1). The numbers in parentheses are the two t-statistics for each coefficient estimate. The top statistic uses uncorrected Fama-MacBeth standard errors; the bottom statistic uses Shanken (1992) correction. The term adjusted R<sup>2</sup> denotes the cross-sectional R<sup>2</sup> statistic adjusted for the degrees of freedom.

Thus overall, the empirical results confirm the theory that option returns help explain securities returns, and imply that investors see options as instruments for hedging against nonnegative wealth levels.

### 3.3 GMM-SDF Estimations

To further check the robustness of the previous findings, we also performed Generalized Method of Moments (GMM) estimations within the Stochastic Discount Factor (SDF) framework outlined in Section 3.3.1. The advantage of a GMM approach is that it allows the estimation of model parameters in a single pass, thereby avoiding the error-in-variables problem in Fama-MacBeth kind of two-pass regressions. Another advantage of GMM, is that it is extremely general in its assumptions and can be applied to all classes of assets.

The set of equations for the method of moments for unconditional versions of C-CAPM is  $E[(1 + R_{j,t+1})m_{t+1}] = 1$ , where  $R_{j,t+1}$  is the net return for a risky asset  $j$ , and  $m_{t+1} = \gamma_0 + \gamma_1 c \widehat{a} y_t + \eta_o^T f_{t+1}$ . The vector of coefficients for fundamental factors is subsets of  $\eta_o = (\eta_{0,\Delta c}, \eta_{0,c_{at}}, \eta_{0,c_{in}}, \eta_{0,c_{out}}, \eta_{0,p_{at}}, \eta_{0,p_{in}}, \eta_{0,p_{out}})$ , and  $f_t$  is as given in i, or ii.

The model for the method of moments for conditional versions of C-CAPM is  $E_t[(1 + R_{j,t+1})m_{t+1}] = 1$ , where  $m_{t+1} = \gamma_0 + \gamma_1 c a y_t + \eta_0^T f_{t+1} + \eta_1^T c a y_t f_{t+1}$ . The vector of coefficients for fundamental factors is the subset of  $\eta_o = (\eta_{0,\Delta c}, \eta_{0,c_{at}}, \eta_{0,c_{in}}, \eta_{0,c_{out}}, \eta_{0,p_{at}}, \eta_{0,p_{in}}, \eta_{0,p_{out}})$ , and the vector of coefficients for

scaled factors is subsets of  $\eta_1 = (\eta_{1,\Delta c}, \eta_{1,c_{at}}, \eta_{1,c_{in}}, \eta_{1,c_{out}}, \eta_{1,p_{at}}, \eta_{1,p_{in}}, \eta_{1,p_{out}})$ .  $f_t$  is as given in iii, or iv.

Table 3.6 presents results of the above GMM-SDF estimations for the 4 models tested. Looking at Row 1, we see that the change in log-consumption does not play a role in constructing the SDF. It has an insignificant coefficient, and furthermore, the pricing errors with this model are significantly different from zero. Row 2 presents the results for the conditional C-CAPM. Consistent with the results of Fama-MacBeth estimations, we can see that the the coefficient of log aggregate consumption scaled with  $cay_t$  is significant, thus it is an important variable in the construction of the SDF. However, although the pricing errors are lower than its unconditional counterpart, they are still significantly different from zero.

Rows 3 and 4 present the SDF coefficient estimates for the unconditional C-CAPM when call and put returns are included, respectively. Consistent with Fama-MacBeth regressions, we see significant coefficients for the returns of at-the-money calls, and out-of money puts. The pricing errors of the model using call returns are lower than its counterpart using put returns. The errors using the HJ weighting matrix are still far from zero, but when the identity matrix is used, pricing errors are within 20 percent limit of not rejecting that they are equal to zero.

**TABLE 3.6**  
**GMM-SDF Estimations**

	Fundamental Factors						Scaled Factors											
	$\gamma_0$	$\gamma_1$	$\eta_{o,\Delta c}$	$\eta_{o,c_{at}}$	$\eta_{o,c_{in}}$	$\eta_{o,c_{out}}$	$\eta_{o,p_{at}}$	$\eta_{o,p_{in}}$	$\eta_{o,p_{out}}$	$\eta_{1,\Delta c}$	$\eta_{1,c_{at}}$	$\eta_{1,c_{in}}$	$\eta_{1,c_{out}}$	$\eta_{1,p_{at}}$	$\eta_{1,p_{in}}$	$\eta_{1,p_{out}}$	<b>HJ dist.</b>	<b>HJ dist. (id)</b>
<b>Row1</b>	1.32 (29.68) (0.00)		-81.21 (-0.59) (0.56)														1.1131 (0.00)	0.0378 (0.72)
<b>Row2</b>	1.29 (14.82) (0.00)	1.90 (1.08) (0.28)	-17.30 (-0.31) (0.76)							-258.62 (-3.39) (0.00)							1.1093 (0.010)	0.0349 (0.75)
<b>Row3</b>	1.50 (3.66) (0.00)		-142.39 (-1.44) (0.16)	0.71 (2.35) (0.02)	0.23 (0.82) (0.42)	0.24 (1.10) (0.28)											0.9680 (0.17)	0.0248 (0.80)
<b>Row4</b>	2.22 (4.69) (0.00)		-358.30 (-1.38) (0.17)				0.70 (1.41) (0.16)	-0.56 (-1.36) (0.18)	-0.57 (-1.81) (0.08)								1.0913 (0.11)	0.0230 (0.82)
<b>Row5</b>	3.00 (3.04) (0.00)	8.31 (1.39) (0.17)	-67.45 (-0.42) (0.68)	1.17 (2.11) (0.04)	0.33 (1.11) (0.27)	0.51 (1.33) (0.19)				-86.71 (-2.64) (0.01)	-52.00 (-1.87) (0.07)	55.37 (0.66) (0.51)	80.53 (1.32) (0.19)				0.7839 (0.24)	0.0205 (0.85)
<b>Row6</b>	2.80 (2.02) (0.05)	13.22 (1.24) (0.22)	-298.75 (-1.25) (0.22)				-0.06 (-0.04) (0.97)	1.75 (1.05) (0.30)	-3.23 (-2.42) (0.02)	-248.74 (-2.56) (0.01)				-15.17 (-0.33) (0.74)	58.43 (1.39) (0.17)	-71.13 (-1.51) (0.14)	0.8459 (0.22)	0.0214 (0.83)

*Note.* This table gives the estimates for the models of moments  $E[(1 + R_{j,t+1})m_{t+1}] = 1$ , and  $E_t[(1 + R_{j,t+1})m_{t+1}] = 1$  for the unconditional and conditional versions of CAPM, respectively.  $R_{j,t+1}$  is the net return for Fama-French's 25 size and book-to-market portfolios, and the data period is 1990:Q1-2006:Q1. For unconditional models,  $m_{t+1} = \gamma_0 + \eta_0^T f_{t+1}$ , where  $\eta_0 = (\eta_{0,\Delta c}, \eta_{0,c_{at}}, \eta_{0,c_{in}}, \eta_{0,c_{out}}, \eta_{0,p_{at}}, \eta_{0,p_{in}}, \eta_{0,p_{out}})$ , or subsets of it, and  $f_t$  is given by i, or ii. For conditional models,  $m_{t+1} = \gamma_0 + \gamma_1 cay_t + \eta_0^T f_{t+1} + \eta_1^T cay_t f_{t+1}$ , where  $\eta_0 = (\eta_{0,\Delta c}, \eta_{0,c_{at}}, \eta_{0,c_{in}}, \eta_{0,c_{out}}, \eta_{0,p_{at}}, \eta_{0,p_{in}}, \eta_{0,p_{out}})$ , and  $\eta_1 = (\eta_{1,\Delta c}, \eta_{1,c_{at}}, \eta_{1,c_{in}}, \eta_{1,c_{out}}, \eta_{1,p_{at}}, \eta_{1,p_{in}}, \eta_{1,p_{out}})$ , or subsets of them, and  $f_t$  is given by iii, iv. The model for the moments are estimated using the GMM approach with the Hansen-Jagannathan weighting matrix. The numbers in parentheses are the t-statistics and their associated p-values respectively. The minimized value of the GMM criterion function is the first item under the "HJ-dist.", with the associated p-values immediately below it. The final column reports HJ-dist. using the identity matrix as suggested by Lettau and Ludvigson (2001).

Next, we check the explanatory power of models which are relevant to the theory outlined here. The results in Rows 5 and 6 present the estimated SDF coefficients for the conditional model using call and put returns, respectively. To summarize, in both versions of the model, at-the-money call, out-of-the-money puts, scaled consumption, and scaled at-the-money calls appear to be significant factors in constructing the SDF. Furthermore, the pricing errors estimated by using the identity matrix are significantly lower than all the other models presented, thus doing a better job in pricing.

Overall, the conditional C-CAPM using option returns outperform all its conditional and unconditional counterparts with or without options, and presents confirmatory evidence regarding the predictions of the theory outlined.

#### **4 CONCLUSION**

In a multiperiod securities markets, where agents are able to trade risky securities at each period in time, we show that options are non-redundant securities due to the nonnegative wealth constraints that agents face in solving their optimal consumption-investment problem. The results are similar to that of Vanden's such that the representative agent holds the aggregate consumption plus options written on the aggregate consumption. However, the contributions are twofold. First on the theoretical side, due to the characteristic of multiperiod reconvening markets, the pricing agent's optimal portfolio leads to a multifactor conditional C-CAPM with option returns as factors. Second, on the empirical side, there have been no tests of asset pricing in the framework of conditional C-CAPM that includes option returns as explanatory variables. The model tested performs better than its conditional and

unconditional counterparts, confirming the theory that option returns should turn up as explanatory variables in securities returns.

Merton stresses that “*the core of financial economic theory is the study of individual behavior of households in the allocation of their resources in an environment of uncertainty and of the role of economic organizations in facilitating these allocations*”. Thus, the theory outlined and the findings presented here have important implications both for improving the allocational efficiency of resources in the economy, for asset pricing, and for capital markets theories.

## APPENDIX

### ***Proof of Proposition 2.***

Assume that the agents' utility function is of the form:

$$u_{i,t}(c_i(a_t)) = c_i(a_t) - \frac{b}{2} c_i^2(a_t)$$

Then the marginal utility and its inverse are given by:

$$u'_{i,t}(c_i(a_t)) = 1 - bc_i(a_t)$$

$$u'^{-1}_{i,t}(c_i(a_t)) = \frac{1}{b}(1 - c_i(a_t))$$

Now define constants,

$$A_1 = \Delta\left(\frac{1}{\gamma_{2,t}}\right), A_2 = \Delta\left(\frac{1}{\gamma_{3,t}}\right), \dots, A_{I-1} = \Delta\left(\frac{1}{\gamma_{I,t}}\right) \quad (\text{A.1})$$

and assume,

$$\gamma_{1,t}^{-1} > \gamma_{2,t}^{-1} > \dots > \gamma_{I,t}^{-1} \quad (\text{A.2})$$

Also define  $\bar{\gamma}_{k,t} = \sum_{j=1}^k \gamma_{j,t}$  .

One can determine the values for the constants,  $A_1, A_2, \dots, A_{I-1}$  by using definition (A.1), and assumption (A.2). For example,

At  $A_1 = \Delta\left(\frac{1}{\gamma_{2,t}}\right)$ , we have;

$$A_1 = \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{1,t}}{\gamma_{2,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{2,t}}{\gamma_{2,t}}\right)\right] + \max\left[0, \frac{1}{b}\left(1 - \frac{\gamma_{3,t}}{\gamma_{2,t}}\right)\right] + \dots$$

According to (A.2), the term inside the first parenthesis is strictly positive, the term inside the second parenthesis is zero, and the terms inside the remaining parentheses are strictly negative. Thus,  $A_1 = \frac{1}{b} \left( 1 - \frac{\gamma_{1,t}}{\gamma_{2,t}} \right)$ .

$A_1$  also satisfies  $\Delta^{-1}(C(a_t))$  in Proposition 2. To see that, note

$$\Delta^{-1}(A_1) = \Delta^{-1} \left( \Delta \left( \frac{1}{\gamma_{2,t}} \right) \right) = \frac{1}{\gamma_{2,t}} = \frac{1 - bA_1}{\gamma_{1,t}}.$$

Similarly, at  $A_2 = \Delta \left( \frac{1}{\gamma_{3,t}} \right)$ , we have;

$$A_2 = \max \left[ 0, \frac{1}{b} \left( 1 - \frac{\gamma_{1,t}}{\gamma_{3,t}} \right) \right] + \max \left[ 0, \frac{1}{b} \left( 1 - \frac{\gamma_{2,t}}{\gamma_{3,t}} \right) \right] + \max \left[ 0, \frac{1}{b} \left( 1 - \frac{\gamma_{3,t}}{\gamma_{3,t}} \right) \right] + \dots$$

Again by using (A.2), terms inside the first and second parentheses are strictly positive, the term inside the third parenthesis is zero, and the terms inside the remaining parentheses are strictly negative. Thus,  $A_2 = \frac{1}{b} \left( 2 - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right)$ .

$A_2$  also satisfies  $\Delta^{-1}(C(a_t))$  in Proposition 2. To see that, note;

$$\Delta^{-1}(A_2) = \Delta^{-1} \left( \Delta \left( \frac{1}{\gamma_{3,t}} \right) \right) = \frac{1}{\gamma_{3,t}} = \frac{2 - bA_2}{\bar{\gamma}_{2,t}}.$$

The rest proceeds similarly, so we can write the predefined constants as;

$$A_1 = \frac{1}{b} \left( 1 - \frac{\gamma_{1,t}}{\gamma_{2,t}} \right), \quad A_2 = \frac{1}{b} \left( 2 - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right), \quad \dots, \quad A_{l-1} = \frac{1}{b} \left( l - \frac{\bar{\gamma}_{l-1,t}}{\gamma_{l,t}} \right)$$

So,  $\Delta^{-1}(C(a_t))$ , is satisfied for the above defined constants.

The derivation of optimal sharing rules is as follows:

For  $0 < C(a_t) \leq A_1$  ;

$$c_1(a_t) = \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{1,t} \left( \frac{1 - bC(a_t)}{\gamma_{1,t}} \right) \right] = C(a_t). \quad (\text{A.3})$$

$$\begin{aligned} c_2(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{2,t} \left( \frac{1 - bC(a_t)}{\gamma_{1,t}} \right) \right] \\ &= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{2,t}}{\gamma_{1,t}} \left( C(a_t) - \frac{1}{b} \right) \right] \\ &= \max \left[ 0, \frac{\gamma_{2,t}}{\gamma_{1,t}} \left( C(a_t) - \frac{1}{b} + \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{2,t}} \right) \right] \\ c_2(a_t) &= \max \left[ 0, \frac{\gamma_{2,t}}{\gamma_{1,t}} (C(a_t) - A_1) \right] = 0. \end{aligned} \quad (\text{A.4})$$

The above expression is equal to zero, due to the fact that the term in the parenthesis is less than or equal to zero for  $0 < C(a_t) \leq A_1$ . Now, by using assumption (2), we can write;

$$c_3(a_t) = \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{3,t} \left( \frac{1 - bC(a_t)}{\gamma_{1,t}} \right) \right] \leq \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{2,t} \left( \frac{1 - bC(a_t)}{\gamma_{1,t}} \right) \right] = c_2(a_t)$$

because  $\gamma_{3,t} > \gamma_{2,t}$ .

Similarly,  $c_4(a_t), c_5(a_t), \dots, c_l(a_t) = 0$ , since  $\gamma_{1,t} > \dots > \gamma_{5,t} > \gamma_{4,t}$ . (A.5)

Next, for  $A_1 < C(a_t) \leq A_2$ ;

$$\begin{aligned}
c_1(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{1,t} \left( \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} \right) \right] \\
&= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - \frac{2}{b} \right) \right] \\
&= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - \frac{1}{b} + \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{2,t}} - \frac{1}{b} \left( 1 + \frac{\gamma_{1,t}}{\gamma_{2,t}} \right) \right) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{2,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) \right] \\
&= \max \left[ 0, A_1 + \frac{\gamma_{1,t}}{\lambda_{2,t}} (C(a_t) - A_1) \right] \\
&= A_1 + \frac{\gamma_{1,t}}{\gamma_{2,t}} [C(a_t) - A_1] \\
&= A_1 + \left( 1 - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} \right) [C(a_t) - A_1]
\end{aligned}$$

$$c_1(a_t) = C(a_t) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} [C(a_t) - A_1]. \quad (\text{A.6})$$

$$\begin{aligned}
c_2(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{2,t} \left( \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} \right) \right] \\
&= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - \frac{1}{b} + \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{2,t}} - \frac{1}{b} \left( 1 + \frac{\gamma_{1,t}}{\gamma_{2,t}} \right) \right) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} + \frac{\gamma_{2,t}}{\gamma_{2,t}} (C(a_t) - A_1) \right]
\end{aligned}$$

$$c_2(a_t) = \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} [C(a_t) - A_1]. \quad (\text{A.7})$$

$$\begin{aligned} c_3(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{3,t} \left( \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} \right) \right] \\ &= \max \left[ 0, \frac{\gamma_{3,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - \frac{1}{b} \left( 2 - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right) \right) \right] \\ c_3(a_t) &= \max \left[ 0, \frac{\gamma_{3,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_2) \right] = 0. \end{aligned} \quad (\text{A.8})$$

Again, by using assumption (2), we can write;

$$c_4(a_t) = \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{4,t} \left( \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} \right) \right] \leq \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{3,t} \left( \frac{2 - bC(a_t)}{\bar{\gamma}_{2,t}} \right) \right] = c_3(a_t)$$

because  $\gamma_{4,t} > \gamma_{3,t}$ .

$$\text{Similarly, } c_5(a_t), c_6(a_t), \dots, c_l(a_t) = 0 \text{ since } \gamma_{l,t} > \dots > \gamma_{6,t} > \gamma_{5,t}. \quad (\text{A.9})$$

Next, for  $A_2 < C(a_t) \leq A_3$ ;

$$\begin{aligned} c_1(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{1,t} \left( \frac{3 - bC(a_t)}{\bar{\gamma}_{3,t}} \right) \right] \\ &= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} \left( C(a_t) - \frac{3}{b} \right) \right] \\ &= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} \left( C(a_t) - \frac{2}{b} + \frac{1}{b} \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} - \frac{1}{b} \left( 1 + \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right) \right) \right] \end{aligned}$$

$$\begin{aligned}
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - A_2 + \frac{1}{b} + \frac{1}{b} \left( -\frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} + \frac{\gamma_{1,t}}{\gamma_{2,t}} \right) \right) \right] \\
&= \max \left[ 0, \frac{1}{b} \left( 1 - \frac{\gamma_{1,t}}{\gamma_{3,t}} + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} - \frac{\gamma_{2,t}}{\gamma_{3,t}} \right) + \left( 1 - \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} + \frac{\gamma_{1,t}}{\bar{\gamma}_{3,t}} \right) (C(a_t) - A_2) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) \right]
\end{aligned}$$

After some rearranging,

$$c_1(a_t) = C(a_t) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) - \frac{\gamma_{1-1} \gamma_{3,t}}{\bar{\gamma}_{2,t} \bar{\gamma}_{3,t}} (C(a_t) - A_2). \quad (\text{A.10})$$

$$\begin{aligned}
c_2(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{2,t} \left( \frac{3 - bC(t)}{\bar{\gamma}_{3,t}} \right) \right] \\
&= \max \left[ 0, \frac{1}{b} + \frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}} \left( C(a_t) - \frac{2}{b} + \frac{1}{b} \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} - \frac{1}{b} \left( 1 + \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right) \right) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{2,t}}{\gamma_{3,t}} + \frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{2,t}}{\gamma_{3,t}} + \frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) + \frac{\gamma_{2-1}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) \right] \\
&= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \frac{\gamma_{2,t}}{\gamma_{3,t}} + \frac{\gamma_{2,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_2) - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} \left( C(a_t) - A_2 + \frac{1}{b} + \frac{1}{b} \left( \frac{\gamma_{1,t}}{\gamma_{2,t}} - \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right) \right) + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) \right] \\
&= \max \left[ 0, \frac{1}{b} \left( 1 - \frac{\gamma_{2-1}}{\gamma_{3-1}} - \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} - \frac{\gamma_{1,t}}{\bar{\gamma}_{2,t}} + \frac{\gamma_{2,t}}{\gamma_{3,t}} \right) + \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) - \frac{\gamma_{2,t} \gamma_{3,t}}{\bar{\gamma}_{2,t} \bar{\gamma}_{3,t}} (C(a_t) - A_2) \right] \\
c_2(a_t) &= \frac{\gamma_{2,t}}{\bar{\gamma}_{2,t}} (C(a_t) - A_1) - \frac{\gamma_{2,t} \gamma_{3,t}}{\bar{\gamma}_{2,t} \bar{\gamma}_{3,t}} (C(a_t) - A_2). \quad (\text{A.11})
\end{aligned}$$

$$\begin{aligned}
c_3(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{3,t} \left( \frac{3 - bC(a_t)}{\bar{\gamma}_{3,t}} \right) \right] \\
&= \max \left[ 0, \frac{1}{b} + \frac{\delta_{3,t}}{\bar{\gamma}_{3,t}} \left( C(a_t) - \frac{2}{b} + \frac{1}{b} \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} - \frac{1}{b} \left( 1 + \frac{\bar{\gamma}_{2,t}}{\gamma_{3,t}} \right) \right) \right] \\
c_3(a_t) &= \frac{\gamma_{3,t}}{\bar{\gamma}_{3,t}} [C(a_t) - A_2]. \tag{A.12}
\end{aligned}$$

$$\begin{aligned}
c_4(a_t) &= \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{4,t} \left( \frac{3 - bC(a_t)}{\bar{\gamma}_{3,t}} \right) \right] \\
&= \max \left[ 0, \frac{\gamma_{4,t}}{\bar{\gamma}_{3,t}} \left( C(a_t) - \frac{1}{b} \left( 3 - \frac{\bar{\gamma}_{3,t}}{\gamma_{4,t}} \right) \right) \right] \\
c_4(a_t) &= \max \left[ 0, \frac{\gamma_{4,t}}{\bar{\gamma}_{3,t}} (C(a_t) - A_3) \right] = 0 \tag{A.13}
\end{aligned}$$

By using assumption (2), we can write;

$$c_5(a_t) = \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{5,t} \left( \frac{3 - bC(t)}{\bar{\gamma}_{3,t}} \right) \right] \leq \max \left[ 0, \frac{1}{b} - \frac{1}{b} \gamma_{4,t} \left( \frac{2 - bC(t)}{\bar{\gamma}_{3,t}} \right) \right] = c_4(a_t)$$

because  $\gamma_{5,t} > \gamma_{4,t}$ .

$$\text{Similarly, } c_6(a_t), c_7(a_t), \dots, c_l(a_t) = 0 \text{ since } \gamma_{l,t} > \dots > \gamma_{7,t} > \gamma_{6,t} \tag{A.14}$$

The rest proceeds in a similar manner. We observe that the equilibrium condition  $\sum_{i=1}^l c_i(a_t) = C(a_t)$  is satisfied for all possible levels of aggregate consumption. Furthermore, the nonnegative wealth constraint of the agent indexed with  $i = 1$  never binds in any of the possible aggregate consumption levels. Thus, we call this as the representative agent, and using A.3, A.6, and A.10, in general, its optimal sharing rule can be written as

$$c_1(a_t) = C(a_t) - \sum_{j=1}^{t-1} \frac{\gamma_{1,t} \gamma_{j+1,t}}{\bar{\gamma}_{j,t} \bar{\gamma}_{j+1,t}} \max[0, C(a_t) - A_j].$$

This completes the proof. ■

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