Consider a supply chain in which a product must pass through multiple sites located in series before it is finally delivered to outside customers. Incentive problems may arise in this system when decisions are delegated to corresponding site managers, each maximizing his/her own performance metric. From the overall system’s point of view, the decentralized supply chain may not be as efficient as the centralized one. In practice, alternative performance mechanisms are often used to align the incentives of the different managers in a supply chain. This paper discusses the cost conservation, incentive compatibility, and informational decentralizability properties of these mechanisms. In particular, for a special type of supply chain, we show that a performance measurement scheme involving transfer pricing, consignment, shortage reimbursement, and an additional backlog penalty at the last downstream site satisfies all these properties.

1. Introduction

Consider a supply chain in which a product passes through multiple sites located in series before it is finally delivered to outside customers. Such a supply chain resembles a serial multi-echelon inventory system whose optimal control policy was characterized in the seminal work by Clark and Scarf (1960). Their analysis takes the perspective of a central planner (or headquarters) who has access to the status of the inventories at all sites and makes all stocking decisions for the entire system. While this single-person optimization paradigm provides valuable insights, in practice, supply chains with multiple inventory sites often operate in a decentralized mode.

In a decentralized decision structure, a site manager is assigned to a single-inventory site and held accountable for a specified set of activities and decisions for the site. Each site operates as a responsibility center (Horngren and Foster 1991), and each site manager is evaluated based on prespecified performance measurement rules, or performance metrics.

Decentralization of decisions rights is an inevitable facet of managing a large organization. Modern organizations constantly face the challenge of making timely decisions using specialized information. An effective way of meeting such a challenge is to delegate decision rights to the “person on the spot” who has the intimate knowledge of his or her immediate surroundings (Hayek 1945). One problem of decentralization is, however, the potential incentive misalignment between the “principal” (the delegating party) and “agents” (the delegates).

One way to mitigate the general problem of incentive misalignment in a decentralized supply chain is through a set of corporate rules such as accounting methods, transfer pricing schemes between sites, performance metrics for site managers, and/or various operational constraints. We call such a set of rules a (performance) measurement scheme. A measurement scheme specifies how sites are to pay each other for transfer of goods, what performance metric should be applied to each individual manager, and what
constraints he or she should abide by. (A similar argument holds for a supply chain comprising independent organizations. The difference is that a performance measurement scheme appears in the form of contractual relationship, as opposed to internal rules.)

As an example, suppose that each site manager in a multisite supply chain in series is evaluated in terms of the inventory and shortage cost at the site. No site manager but the last downstream manager directly faces stockout costs for not filling a customer order on time. Thus, upstream managers have little incentive to carry buffer inventories. In expectation of this, the last downstream manager has to carry extra inventories to buffer against demand uncertainties and replenishment leadtimes. From the overall system’s point of view, this is usually an inefficient policy, since final goods incur the highest inventory holding costs.

The present paper examines the problem of specifying a measurement scheme that could help to align the incentives of the multiple site managers so that the overall supply chain efficiency can be enhanced.

2. On Performance Measures in Decentralized Supply Chains

A variety of arrangements are observed in practice to mitigate the incentive problem described in the last section. One commonly observed approach is to enforce a certain fill-rate target to the upstream manager so that the downstream manager may be guaranteed on-time deliveries for a fraction of its orders. A typical performance metric is some function of the actual operation cost and the fill-rate achieved at the site.

Another approach is to impose a penalty on each stock-out. According to Chandler (1990), for example, Sears Roebuck had multiple upstream departments at the turn of the century, each of which managed inventories of a set of goods filling orders from the mail-order processing department. If a certain upstream department failed to deliver a requested item within fifteen minutes to the assembly room, the negligent department was charged a penalty of fifty cents per item, as well as absorbing the extra shipping cost. Here, the penalty function is linear in unfilled demand. In this case each site manager may be evaluated in terms of full site cost which is the sum of operation costs and internal penalties. Both these approaches are intended to mitigate the incentive problems in decentralized inventory control systems.

What is a “good” performance measurement scheme? We briefly discuss three viable properties of a scheme in the present setting. One is the cost conservation property. It says that the accounting system built into a scheme should trace all costs to individual sites, and thus does not require any subsidies or taxes from headquarters. This is highly desirable, since it is simpler to implement and will not distort the information on the costs of the operation. In particular, this property becomes critical in a supply chain partnership where sites belong to different corporations. In this case, headquarters does not exist to break the budget constraint, and it would be hard to have a neutral party paying subsidies or collecting taxes.

Incentive compatibility is the core property of a good measurement scheme. A scheme is called incentive compatible if it completely eliminates potential incentive misalignment problems. That is, under such a scheme each manager finds it to his or her interest to follow the optimal decision rules for the system as a whole.

The last viable property of a measurement scheme is informational decentralizability, meaning that the scheme can be implemented with site information only, without inventory information at the other sites (such as required in echelon inventory). While decisions using echelon inventory information cannot be worse than using site information only, the incremental benefit should be traded off with the cost of implementing information sharing. The cost not only includes the cost of information systems but also the time and effort to reach an agreement among site managers. Informational decentralizability eliminates the cost of information sharing among sites, and thus is a viable property.

Research on incentive and information problems originates in economics (see, for example, Wilson 1968, Ross 1973, Groves and Loeb 1975, Holmstrom 1979, and Myerson 1979), and has been applied to various functional fields, including accounting (Baiman 1982), finance (Jensen and Meckling 1976) and marketing (Basu et al. 1985).

In the field of Operations Management, there are at
least two lines of incentive study: inventory control and quantity discount. In the context of inventory control, incentive issues are raised by Ackoff (1967), and later analyzed by Porteus and Whang (1991). These papers reconstruct the newsvendor problem in a multiperson framework, where demand information is privately held by the marketing manager while the inventory decision is made by the manufacturing manager. Conflicts of interest arise, since the manufacturing manager concerned with cost minimization tries to minimize inventory-carrying costs, while the marketing manager concerned with revenue maximization tries to minimize stockouts. Ackoff (1967) reports a sequence of events that may arise as a result of incentive and information problems. Porteus and Whang (1991) develop an incentive-compatible mechanism which includes an internal futures market.

The incentive issue related to quantity discount in a two-echelon system has been considered by Monahan (1984). The system manager can offer a price discount (that is independent of the order quantity) to induce the downstream manager to order in quantities that result in cost improvement for the upstream manager. Monahan’s work has sparked a number of follow-on studies (see Banerjee 1986, Lee and Rosenblatt 1986, and Dada and Srikanth 1987). Rosenblatt and Lee (1985) study a similar problem, but focus on quantity discount as the incentive mechanism. These works, however, do not consider demand uncertainties or more general performance measurement schemes. Moreover, since demands are constant and deterministic, the informational aspects of the problem do not arise.

Federgruen and Zipkin (1984) find that, by constructing cost functions appropriately, a decentralized system can perform equally as a centralized one for a multi-echelon inventory system with a single retailer, and can perform almost as well as a centralized one with multiple retailers. This observation is very insightful, and indeed is corroborated by our results in the following section.

3. A Multi-Echelon Supply Chain Revisited

Consider an infinite horizon, two-echelon inventory model with positive replenishment delays, zero setup costs, and random external demands. All the qualitative results will remain valid for a system with an arbitrary number of stocking sites in series, although the analytical development is more tedious and complicated.

A1. (Multi-Echelon) The two stocking sites are indexed by \( m = 1, 2 \). Receiving intermediate product from the upstream Site 2, the downstream Site 1 adds value and holds it until it sells to outside customers. Site 1 faces random external periodic demands, which are identically and independently distributed. To simplify notation, Site 3 denotes the external source of the product, and Site 0 denotes the outside end customer. The external source carries an infinite amount of inventory. Unmet demands at Site 1 are backordered.

A2. (Leadtime) There is a constant leadtime of one period to replenish Site \( m \) from Site \((m + 1)\)’s stock for \( m = 1, 2 \). This leadtime includes both shipment and processing delays. The results can easily be extended to arbitrary fixed leadtimes.

A3. (Timing of Events) At the beginning of a period, replenishments that are due to arrive in that period arrive, and then demand occurs. At the end of the period, inventory and shortage costs are charged against stocks and/or shortages, followed by ordering decisions and the initiation of shipments. Site 1 first orders from Site 2, which then orders from Site 3.

A4. (Costs) Site \( m (m = 1, 2) \) incurs a unit cost of \( c_m \) (and zero setup cost) to order items from Site \( m + 1 \). An inventory holding cost of \( h_m \) is incurred at Site \( m \) per unit of inventory in each period. Further, \( h_1 > h_2 \); i.e., as the product moves along the supply chain, holding cost increases. Pipeline inventory cost in transit to \( m - 1 \) is assessed as part of stock at \( m \). Shortage cost \( \pi \) per unit is incurred only at Site 1 in each period.

All cost parameters are stationary.

Define the following parameters, which are observed at the costing point between the realization of demands and ordering/shipping:

- \( a \) = discount factor for costs per period;
- \( \xi \) = demand in a period;
- \( u_m \) = on-hand inventory level at Site \( m \) (for site 1, negative \( u_t \) denotes backlog);
- \( w_m \) = amount in transit from Site \( m + 1 \) to Site \( m \);
- \( x_w \) = echelon inventory position at Site \( m \);
\[ v_m = \text{echelon inventory level at Site } m, \text{ and} \]
\[ y = \text{inventory position of Site 1 at the end of period} \]
\[ z = \text{amount ordered in a period at Site 2; and} \]
\[ L_m(u) = \text{inventory and shortage cost incurred at Site} \]
\[ m \text{ in a period with on-hand inventory } u, \text{ and} \]
\[ L_m(u) = h_2u; L_1(u) = h_1u^* + \pi(-u)^+, \text{ where } u^* = \max(u, 0). \]

Assuming the entire system as controlled by a single person (the central planner or headquarters), Clark and Scarf (1960) characterize the optimal policy for finite planning horizons. Federgruen and Zipkin (1984) extend and simplify these results for the case of infinite horizons. The optimal policy states that there exist order-up-to levels \((S^*_1, S^*_2)\) for each period, so that the optimal policy at each Site \(m\) is to order from the upstream Site \(m+1\) to bring its echelon inventory position up to \(S^*_m\) as closely as possible. If Site \(m\)'s echelon inventory position exceeds \(S^*_m\), then no order is issued.

The solution \((S^*_1, S^*_2)\) is first-best, in the sense that all relevant information is available to the central planner, and it assumes away potential incentive misalignments. Accordingly, the solution serves as a lower bound to the cost achievable by any decentralized system.

Consider now a decentralized inventory system in which Site Manager \(m\) (=1, 2) is in charge of Site \(m\), making all ordering decisions at his or her site to optimize a given performance metric, which is specified by a measurement scheme.

We now introduce a specific measurement scheme \(A\) that is characterized by:

1. (Transfer Pricing) For each unit ordered by Site 1, Site 1 pays Site 2 the actual variable cost \(c_2\). Thus, Site 2 makes zero profit. (If contrarily a profit is added to the transfer price, Manager 1 will face a bloated unit price and make suboptimal quantity decisions—a phenomenon known as the price distortion effect, see Hirshleifer (1964).)

2. (Consignment) Site 2 pays the holding cost of its intermediate product in Echelon 2 (i.e., at Sites 1 and 2) at rate of \(h_2\). Site 1 pays \(h_1\) for its inventory. Since Site 2's echelon inventory without counting backlogs is \(v_2 - v_1 + (v_1)^+\), Site 2's share of inventory cost is \(h_2[v_2 - v_1 + (v_1)^+]\). One way to implement this is through a consignment arrangement as follows: Site 1 pays the transfer price only when the product is sold to outside customers, and pays other costs associated with carrying inventory at the lower site as well as at its own. Alternatively, headquarters pays Site 2 at the time of sales, while Site 1 receives the intermediate product free from Site 2. In the latter implementation, transfer pricing is between headquarters and the sites.

3. (Additional Backlog Penalty at Site 1) The consignment deal is void for orders backlogged at Site 1. If Site 1 is in a backorder position (i.e., \(v_1 < 0\)), Manager 1 pays a unit cost \(h_2\) for the units backordered at Site 1, in addition to the backlog cost \(\pi\).

This additional backlog penalty \(-h_2v_1\) is paid to Site 2. Based on (2) and (3), the cost charged to Site 2 is

\[ \tilde{L}_2(v_2) := h_2[v_2 - v_1 + (v_1)^+] - h_2(-v_1)^+ = h_2(v_2 - v_1) + \tilde{h}_2v_1 = \tilde{h}_2v_2. \]

For Site 1, the total cost is:

\[ \tilde{L}_1(v_1) := h_1(v_1)^+ + (\pi + h_2)(-v_1)^+ = h_1v_1 + (\pi + h_2)(-v_1)^+. \]

4. (Shortage Reimbursement) Site 2 reimburses Site 1 if it fails to deliver the ordered quantity completely. Suppose Manager 1 requests an order in a period to bring the echelon inventory position to \(y\), but Manager 2 has echelon stock of \(v_2\ (\leq y)\), and thus Manager 1's request cannot be immediately shipped in full. Then, Manager 2 pays \(G(v_2, y)\) to Manager 1, where

\[ G(v_2, y) := \begin{cases} -(1 - \alpha)c_1(y - v_2) + \tilde{L}^2_m(v_2) - \tilde{L}_2^2(y), & \text{if } v_2 < y, \\ 0, & \text{if } v_2 \geq y; \end{cases} \]

and \(\tilde{L}^2_m(x) := \alpha^mE[L_m(x - \xi(m))], m = 1, 2; k = 1, 2, \cdots\) with \(\xi(m)\) denoting a random variable whose distribution is given by the \(k\)-fold convolution of \(\xi\).

Under \(A\), therefore, Manager 1 takes Manager 2's strategy \(\mathcal{P}^*_2\) as given and solves:

\[ C^*_1(v_1, v_2, w_1, w_2 | \mathcal{P}^*_2) = \min_{y, y^*} C^*_1(y | v_1, v_2, w_1, w_2, \mathcal{P}^*_2), \quad (3.1) \]
where

\[ C_1(y | v_1, v_2, w_1, w_2, \mathcal{P}_2^*) \]

\[
\begin{cases}
  c_1(y - x_1) + \bar{L}_1(v_1) + \alpha E[C_1^*(v_1 + w_1 - \xi, v_2 + w_2 - \xi, y - x_1, w_2^1 | \mathcal{P}_2^*)], & \text{if } v_2 > y; \\
  c_1(y - x_1) + \bar{L}_1(v_1) + c(y - v_2), & \text{if } v_2 \leq y,
\end{cases}
\]

(3.2)

where \( w_2^1 \) is the new \( w_2 \) as specified by \( \mathcal{P}_2^* \).

Likewise, given Site 1’s optimal strategy, Manager 2’s problem is:

\[
C_2^*(v_2, w_2 | \mathcal{P}_1^*)
\]

\[
= \min_{z \geq 0} \left( c_2 z + \bar{L}_2(v_2) + G(v_2, \mathcal{P}_1^*) + \alpha E[C_2^*(v_2 + w_2 - \xi, y, z | \mathcal{P}_1^*)] \right),
\]

(3.3)

where \( \mathcal{P}_1^* \) is the optimal inventory strategy chosen by Manager 1.

**Proposition 1.** Under the measurement scheme \( \hat{A} \), the total cost of the system is fully allocated to site managers in each period.

The proposition implies that the accounting system built into \( \hat{A} \) traces all costs to individual sites and thus does not require any subsidy or tax from headquarters. Hence, \( \hat{A} \) satisfies the cost conservation property, and in fact, the total operating cost in each period is fully allocated to site managers, i.e., the cost conservation property applies in each period.

**Proposition 2.** The performance measurement scheme \( \hat{A} \) is incentive-compatible.

The proof of Proposition 2 (in the Appendix) also indicates that \( \mathcal{P}_1^* \) and \( \mathcal{P}_2^* \) are both characterized by base-stock policies; i.e., there exist \( S_1^* \) and \( S_2^* \), similar to Clark and Scarf (1960), for Sites 1 and 2 respectively, that are optimal order-up-to levels.

The shortage reimbursement \( G \) may be interpreted as delivery warranty, since it exactly covers the losses when orders from Manager 1 are not filled completely.

Note that \( G(v_2, y) = \bar{l}_2^*(v_2) - \bar{l}_2^*(y) - c_1(1 - \alpha)(y - v_2) \) when \( y > v_2 \). Hence, when Site 2 cannot meet the full order of Site 1, it pays Site 1 the expected loss to Site 1 due to the deficit, i.e., \( \bar{l}_2^*(v_2) - \bar{l}_2^*(y) \). This payment, however, is reduced by \( c_1(1 - \alpha)(y - v_2) \), which represents the discounted value of the cost that Site 1 can now defer for not having received (and therefore paid for) \( y - v_2 \) units from Site 1 right away.\(^1\)

Although incentive-compatible, the measurement scheme \( \hat{A} \) requires echelon-level decision making, i.e., Site 2’s inventory decisions are based on echelon inventory information. This requires that a manager should keep track of his or her echelon inventory. From Axsater and Rosling (1993), we can see that \( \hat{A} \) can be implemented with each manager knowing his or her site inventory position only (not echelon inventory position).

**Proposition 3.** Consider an “informationally decentralized” inventory control system in which each individual site operates as an order-up-to system in the following manner: If site inventory position at \( m = 1, 2 \) is less than \( S_m^* - S_{m-1}^* \) (with \( S_0^* := 0 \)), then make a request to site \( m + 1 \) to bring the site inventory position at \( m \) up to \( S_m^* - S_{m-1}^* \); otherwise, order nothing. Deliveries are made as much as possible with the balance fully backlogged. The above decentralized system would achieve the first-best solution. As a result, \( \hat{A} \) is incentive-compatible and can be implemented with site-specific information only.

The cost conservation and incentive-compatibility properties of \( \hat{A} \) can be extended to the case when demands are nonstationary (whose details are available from the authors). But \( \hat{A} \) may not be always informationally decentralizable in that case. This implies that information sharing as well as an incentive mechanism must be arranged to attain the first-best solution under nonstationary demands.

4. Concluding Remarks

Decentralized implementation of supply chains requires the development of performance measurement schemes to align the incentives and interests of the

\(^1\) An earlier draft of the present paper (Lee and Whang 1992) assumes a finite planning horizon model with nonhomogeneous costs. It demonstrates an equivalent result using a slightly different version of \( \hat{A} \), according to which site 2 pays site 1 the actual loss incurred due to a deficit.
multiple managers in the supply chain. We describe some desirable properties of performance measurement schemes: the cost conservation, incentive compatibility, and informationally decentralizability. Interestingly, a performance measurement scheme $\tilde{A}$ satisfying such properties can be derived for the specific multi-echelon inventory system studied by Clark and Scarf (1960).

The implementation of $\tilde{A}$ requires a set of assumptions and setups. For one thing, the distribution of final demands has to be commonly known to all site managers. One plausible scenario is that headquarters (with the help of marketing) perform demand forecasting for site managers. Based on the forecasted demand distributions, each manager observes his or her inventory position, decides the order quantity, and pulls from the upstream site. Each manager minimizes the full site cost ($= \text{sum of operation cost and internal prices as prescribed by $\tilde{A}$}$). Although not part of our model, managers may be compensated according to the performance measured in terms of the realized objective value. In making decisions, each manager understands that the other manager also makes decisions to minimize his or her own objective as a competent optimizer. Once the measurement scheme $\tilde{A}$ is adopted and announced to managers, headquarters need not make any ordering decisions.

The scheme $\tilde{A}$ has several intriguing characteristics. First, the consignment arrangement asks Manager 1 to pay only a portion of the holding cost, while the remaining cost is charged to Manager 2. Thus, the scheme distorts the inventory cost structure faced by Manager 1. The impact of consignment on inter or intrafirm coordination has received little attention in inventory theory, perhaps because consignment is simply viewed as a mode of payment carrying no operational or economic implications. The present work, by contrast, underscores how consignment can change the inventory cost structure facing site managers and how it affects their inventory decisions.

Another important aspect of the proposed scheme is shortage reimbursement to motivate the upstream manager to carry inventories. Other popular organizational schemes such as fill-rate targets or linear penalties may be viewed as substitutes for our more sophisticated penalty $G$.

While both consignment and shortage reimbursement favor Manager 1 by spreading the burden of carrying inventory across sites, the scheme also imposes an additional backlog penalty on Manager 1 for the units backlogged at Site 1. This backlog penalty induces Manager 1 to carry inventory.

The scheme $\tilde{A}$ is informationally decentralizable for stationary demands. It can be shown that a decentralized inventory system can achieve the first-best outcome for a product in a growing or steady market. But if one anticipates a decline or fluctuation in demand over time (possibly near the end of the product life cycle or due to seasonality in demand), echelon-level coordination will yield strictly better performance than any informationally decentralized system.

The model assumes that the sites lie within a single organization. But certain incentive problems and solution concepts (such as consignment and shortage reimbursement) may also apply to a system that spans multiple organizations. For example, GM Saturn’s purchasing contract with its part suppliers is such that Saturn pays upon usage of the parts, rather than on delivery. Furthermore, Saturn recently announced a penalty scheme, whereby its part suppliers are fined $500 for each minute of Saturn’s lost production caused by their quality or delivery problems (USA Today 1992).

Hammond (1990) states that many apparel and textile manufacturers are skeptical about the benefits of Quick Response programs, while downstream retailers are more receptive to the programs. The manufacturers’ major concern is that the programs may shift inventories upstream. This is plausible in the present absence of special arrangements such as delivery warranties or consignment. While Hammond (1990) describes a technological solution, the present paper suggests that re-alignment of contractual relationships among the participants may improve the overall efficiency of the system.

From the above discussion, it is clear that when multiple organizations are involved in a supply chain, performance schemes are much more complex to develop. For one thing, consensus may be harder to
reach among participants over the implementation of a new scheme, since it affects the payoff structures of different sites, and no central governing entity exists to absorb the resulting differentials. Our measurement scheme $\hat{A}$ has the nice property in that there is no need for subsidies or taxes from headquarters.

An important question that one may ask is, why decentralize? One common reason is that local managers hold local information not available to the headquarters, and therefore they are relied upon to make some decisions. Under such information asymmetry, a decentralized policy with an appropriate incentive mechanism to coordinate the local managers can strictly dominate centralized decision-making without access to local information. Design of such a mechanism with information asymmetry is beyond the objective of the present paper, but is a good topic for future research. An example of work along this line includes Riordan (1984), who studies the mechanism design in a bilateral monopoly with uncertainty and asymmetric information. Suppose that two managers located sequentially in a supply chain are involved in a one-shot transactional relationship. Site Manager 1 privately knows the production cost, while Manager 2 has private information about the market demand. Thus, the optimal centralized policy cannot be derived or implemented due to the lack of information. The objective of the mechanism design (or performance metrics) is to induce managers to share their private information honestly and choose the optimal order quantity based on the private information revealed. Riordan (1984) proposes a class of decentralized mechanisms that operate in three steps: (a) Manager 1 observes the production cost and announces a transfer payment schedule to Manager 2; (b) Manager 2 observes private information on market demand and places an order; and (c) the ordered quantity is delivered to Manager 2, who pays according to the transfer payment schedule. Riordan specifies the pair of payment schedule and order quantity (each given as a formula involving the two parameters revealed by managers) that in equilibrium achieves the first-best outcome.

Finally, are there other decentralized schemes that are incentive compatible? If so, how would we compare among different decentralized schemes? In fact, there exist other incentive-compatible decentralized schemes; for example, charge each site manager a fixed percentage of the total system-wide cost. Then, each manager will share the same system objective, and the first-best outcome will be attained in equilibrium. Accounting will be simpler then, since it requires only system-wide cost information, and not site-specific information. To address these and other interesting questions, we hope that a richer model will be developed in future research.  

Appendix

Proof of Proposition 1. From the definition of $x_m$ and $v_{m1}$, $v_2 = u_m + x_m$ and $x_m = v_\pi + w_m$ $(m = 1, 2)$. Thus, $v_2 = u_2 + w_1 + v_1$. Hence,

$$L_1(v_1) + L_2(v_2) = h_1v_1 + h_2v_2 + (\pi + h_1)(-v_1)$$

$$= h_2(v_2 - v_1) + h_1v_1 + (\pi + h_1)(-v_1)$$

$$= h_2(u_2 + w_1) + h_1(v_1) + \pi(-v_1)$$

$$= L_1(v_1) + L_2(u_2 + w_1). \quad \Box$$

Proof of Proposition 2. We offer here a sketchy proof, referring interested readers to Lee and Whang (1992) for a more rigorous proof. (Its proof is for the finite horizon case, but one can apply the techniques of Federgruen and Zipkin (1984) to extend the result to the infinite horizon case.) Clark and Scarf (1960) showed that the optimal centralized system would have site 1 operating under the base stock policy, where the base stock is determined as if site 2 has infinite supply. Hence, this is equivalent to solving a single location problem characterized by the following dynamic program:

$$f_1(v_1, w_1) = \min_{y > 0} \{ c_1(y - x_1) + \tilde{L}_1(v_1)$$

$$+ \alpha E[f_1(v_1 + w_1 - x_1, y - x_1)] \}. \quad (A.1)$$

For site 2, Clark and Scarf showed that the problem can be stated as:

$$f_2(v_2, w_2) = \min_{z > 0} \{ c_2z + \tilde{L}_2(v_2) + \Gamma(v_2)$$

$$+ \alpha E[f_2(v_2 + w_2 - x_2, z)] \}. \quad (A.2)$$
from (A.3),

\[
\begin{align*}
\alpha E[f(v_1 + w_1 - \xi, y - x_1) - f(v_1 + w_1 - \xi, v_2 - x_1)]
&= \alpha [\hat{E}[y - \hat{y}] + g_1(y - \hat{y}) - \hat{I}_1(v_2 - \hat{y}) - g_1(v_2 - \hat{y})] \\
&= \hat{I}_1(y - \hat{y}) - \hat{I}_1(v_2) + \alpha E[g_1(y - \hat{y}) - g_1(v_2 - \hat{y})] \\
&= \hat{I}_1(y - \hat{y}) - \hat{I}_1(v_2) - \alpha c(v_2 - y),
\end{align*}
\]

Hence, the case of \( v_2 < y \) in (A.4) can be rewritten as

\[
c_1(y - x_1) + \hat{L}_1(v_1) + \alpha E[f_1(v_1 + w_1 - \xi, y - x_1)],
\]

which is equivalent to the case of \( v_2 \geq y \) in (A.4) and is also equivalent to \( f(v_1, w_1) \) from (A.1). Therefore, (3.1) becomes equivalent to (A.1) when substituting \( f(v_1, w_1) \) for \( C_2(v_1, v_2, w_1, w_2 | \hat{S}^*_2) \). This result shows that the order-up-to-\( S^*_2 \) policy is the optimal solution to (A.4). Note that \( S^*_2 \) is a dominant strategy regardless of the strategy chosen by site 2, all due to the specific measurement scheme A used.

Also note that \( G(v_1, S^*_2 | \hat{S}^*_2) = \Gamma(v_2) \) in (3.3) and (A.2). Thus, \( C_2(v_1, v_2, w_1, w_2 | \hat{S}^*_2) = f(x_2, y_2) \).

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