Optimal oblivious routing under statistical uncertainty*

Pietro Belotti[1] and Mustafa Ç. Pınar[2]

[1]Dip. di Elettronica ed Informazione, Politecnico di Milano, Italy.
[2]Dept. of Industrial Engineering, Bilkent University, Ankara, Turkey.

Abstract. In telecommunication networks, a common measure is the maximum congestion (i.e., utilisation) on edge capacity. As traffic demands are often known with a degree of uncertainty, network management techniques must take into account traffic variability. The *oblivious performance* of a routing is a measure of how congested the network may get, in the worst case, for one of a set of possible traffic demands.
We present two models to compute, in polynomial time, the optimal oblivious routing: a linear model to deal with demands bounded by box constraints, and a second-order conic program to deal with statistical uncertainty, i.e., when a mean-variance description of the traffic matrices is given. A comparison between the optimal oblivious routing and the well-known OSPF routing technique on a set of real-world networks shows that, for different levels of uncertainty, optimal oblivious routing has a substantially better performance than OSPF routing.

Keywords: Traffic engineering, oblivious routing, linear programming, second order cone programming.

1 Introduction

Telecommunication networks are an important infrastructure of today's economy; the cost of connecting a community through wired or wireless technologies is paid off by the benefits offered by rapid data transfer. However, the variety of technologies available and the complexity of such invasive structure pose difficult problems to operators, both in the design and management of networks.

We focus on a class of problems where the link capacity is known in advance, and a set of origin-destination requests of flow, called *traffic demand* (or simply *demand*), is given. Then one faces the problem of *routing* these requests of flow such that the network capacity is not overloaded. Techniques of *traffic engineering* allow for a routing that does not affect the network performance (delay, data loss), thus guaranteeing a certain quality of service.

A common measure of the network usage is the maximum link *congestion* for a given routing, i.e., the maximum percentage of used link capacity. The

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more congested a network, the more prone to instability it is in the event of a change in the traffic requests. It is desirable then to devise a routing that gives the minimum congestion for a given demand. As shown in the next section, this amounts to solving a linear programming (LP) problem.

However, the traffic demand d is seldom known with accuracy, either for difficulties in measuring d or because d can vary in time, and one can suppose that there is a set $\mathcal{D}$ of possible demands. It is useful then to compute the *oblivious performance ratio* of a routing, i.e., the worst-case ratio between the congestion with any $d \in \mathcal{D}$ and the congestion that an alternate routing attains for d.

This paper presents two models that obtain, in polynomial time, the optimal oblivious routing assuming that either the demand has lower and upper bounds, or is described by mean-covariance information. These representations of demand uncertainty are motivated by the presence in the literature of several techniques that estimate d with some level of accuracy, expressed with box constraints or mean-variance distribution [14–16]. Our models are inspired by the min-max robust optimization methodology of [4].

Often, real-world data networks follow the Open, Shortest Path First (OSPF) policy: routes are chosen as shortest paths between origin and destination, where arc weights are chosen depending on network parameters such as edge capacity. As arc weights are the only degree of freedom to play with, routing optimisation consists in finding the value of weights so as to minimise some network performance measure [6, 7, 10]. Other routing techniques such as *Multi-Protocol Label Switching* (MPLS) do not constrain route length, thus allowing for the implementation of any routing. As shown in [7], this greater flexibility pays off in terms of network performance. The second contribution of this work is a comparison between the performance of a finely tuned routing and that of OSPF routing as commonly implemented in today's networks, that shows that the oblivious performance ratio of OSPF routing can be greatly improved.

In the next section we present the routing problem and some additional notation. The concept of oblivious routing is introduced in Section 3, and the two models are presented in Sections 4 and 5. We report some tests on real-world networks in Section 6 and give some conclusions in Section 7.

2 Routing and congestion in telecommunication networks

Consider a network topology defined by an undirected graph $G = (V, E)$ whose edges $e \in E$ are assigned a capacity c_e. An edge e may also be denoted by the set $\{h, k\}$ of its endnodes. Associated with E is the set of directed arcs A containing all pairs (h, k) and (k, h) such that $\{h, k\} \in E$. The *neighbourhood* of node h, i.e., the set of nodes adjacent to h, is defined as $N(h) = \{k \in V : \{h, k\} \in E\}$.

An *origin-destination pair* (o-d pair) is an oriented pair (i, j) of nodes in V requesting an amount of flow d_{ij} to be sent from i to j. Let D be the set of all o-d pairs. A *traffic matrix* $d = (d_{ij})$ is a vector of requests between all $(i, j) \in D$.

The fraction of demand (i,j) flowing on edge $\{h,k\}$ in the direction $h \to k$ is denoted as f_{hk}^{ij}; for the sake of readability, we denote with f_{hk} the vector (f_{hk}^{ij}), $(i,j) \in D$, and with f_e, where $e = \{h,k\} \in E$, the vector $f_{hk} + f_{kh}$.

We denote the total flow on edge e with $\textsc{flow}(e,f,d) = \sum_{(i,j)\in D} d_{ij}(f_{hk}^{ij} + f_{kh}^{ij}) = d^T(f_{hk} + f_{kh}) = d^T f_e$. A *routing* of a demand d is the set of all f_{hk}^{ij} for each o-d pair $(i,j) \in D$ and arc $(h,k) \in A$, such that flow conservation holds:

$$\sum_{k \in N(h)} (f_{hk}^{ij} - f_{kh}^{ij}) = \begin{cases} d_{ij} & \text{if } h = i \\ -d_{ij} & \text{if } h = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall h \in V, (i,j) \in D.$$

A routing is *feasible* if the network capacity can support it, i.e., $\textsc{flow}(e,f,d) \leq c_e$ for all $e \in E$; a demand d is *feasible* when there exists a feasible f for d. The *congestion* is the maximum fraction of capacity used on the graph edges:

$$\textsc{cong}(f,d) = \max_{e \in E} \left(\textsc{flow}(e,f,d)/c_e\right).$$

Let us denote as $\mathcal{F}$ the set of all feasible routings. If the demand d is known a priori, then the routing with the minimum congestion ratio, $\textsc{opt}(d) = \min_{f \in \mathcal{F}} \textsc{cong}(f,d)$, is computed by solving the following linear problem:

$$\textsc{opt}(d) = \min \ z \tag{1}$$

$$\text{s.t.} \qquad z \geq \sum_{(i,j)\in D} (g_{hk}^{ij} + g_{kh}^{ij})/c_e \qquad \forall e = \{h,k\} \in E \tag{2}$$

$$\sum_{k \in N(h)} (g_{hk}^{ij} - g_{kh}^{ij}) = \begin{cases} d_{ij} & \text{if } h = i \\ -d_{ij} & \text{if } h = j \\ 0 & \text{otherw.} \end{cases} \qquad \forall h \in V, (i,j) \in D \tag{3}$$

$$\sum_{(i,j)\in D} (g_{hk}^{ij} + g_{kh}^{ij}) \leq c_e \qquad \forall e = \{h,k\} \in E \tag{4}$$

$$g \geq \mathbf{0}, \tag{5}$$

where we use routing variables g; notice that constraint (4) is equivalent to $z \leq 1$.

3 Demand uncertainty and oblivious routing

From now on, we assume that the traffic matrix d is not known a priori but can be any member of a set $\mathcal{D}$ of traffic matrices. Not surprisingly, the problem gets more difficult as *robust* network management is needed. The problem of routing a set of demands under uncertainty has received much attention recently. Li et al. [9] deal with the *multicast* case, where a demand has one source but multiple destinations. Lin et al. [10] present a Lagrangian Relaxation-based algorithm for the problem where demands are routed on single paths, while Roughan et al. [12] propose a simulation approach to solve both the estimation and the routing problem.

If a routing f and a set $\mathcal{D}$ of possible traffic demands are given, the congestion ratio of f may be defined as the worst-case congestion ratio within $\mathcal{D}$, i.e., $\max_{d\in\mathcal{D}} \textsc{cong}(f,d)$. However, it is more convenient to measure the worst-case ratio, over all $d \in \mathcal{D}$, between $\textsc{cong}(f,d)$ and the best possible congestion for d, $\textsc{opt}(d)$. The *oblivious performance ratio*

$$\textsc{opr}(f,\mathcal{D}) = \max_{d\in\mathcal{D}} \frac{\textsc{cong}(f,d)}{\textsc{opt}(d)}$$

is a measure of the redundancy of f with respect to the demand uncertainty $\mathcal{D}$. In the next sections, we show how to obtain, in polynomial time, the routing with minimum oblivious performance ratio for two different uncertainty figures. Azar et al. [3] present an LP model with possibly exponentially many constraints and a polynomial-time procedure to find the oblivious routing over general networks; Applegate and Cohen [2] present a polynomial size LP model that yields such a routing on the set $\mathcal{D}$ of all demands admitting feasible routing on G. This conservative assumption implies that, for the worst-case demand $\bar{d} = \operatorname{argmax}_{d\in\mathcal{D}} \frac{\textsc{cong}(f,d)}{\textsc{opt}(d)}$, the capacity of at least one edge is totally used, i.e., $\textsc{opt}(\bar{d}) = 1$. Thus, the problem

$$\min_{f\in\mathcal{F}} \max_{d\in\mathcal{D}} \frac{\max_{e\in E} \textsc{flow}(e,f,d)/c_e}{\textsc{opt}(d)} \tag{6}$$

reduces to $\min_{f\in\mathcal{F}} \max_{d\in\mathcal{D}} \max_{e\in E}(\textsc{flow}(e,f,d)/c_e)$; then, by swapping the two *max* operators and by strong duality, a linear programming model is obtained.

The assumption $\textsc{opt}(d) = 1$ is no longer valid if the set of demands is further limited, e.g., by box constraints or within a mean-variance region. In fact, suppose that $\mathcal{D}$ is the set of demands admitting a routing in G and such that $d_{ij} \leq \alpha \frac{\min_{e\in E} c_e}{|D|}$ for all $(i,j) \in D$, with $\alpha < 1$. For all demands $d \in \mathcal{D}$, there is a routing f such that even if all demands were routed on the edge $\bar{e}$ with minimum capacity, $\textsc{flow}(\bar{e},f,d) \leq \alpha c_{\bar{e}}$ and hence $\textsc{opt}(d) \leq \alpha < 1$ for all $d \in \mathcal{D}$.

In order to achieve a valid model, observe that $\textsc{opt}(d)$ does not depend on e, therefore (6) is equivalent to

$$\min_{f\in\mathcal{F}} \max_{e\in E} \max_{d\in\mathcal{D}} \frac{\textsc{flow}(e,f,d)/c_e}{\textsc{opt}(d)}$$

(notice that we have swapped the *max* operators). The model is as follows:

$$\begin{aligned} & \min\ r \\ \text{s.t.} \quad & r \geq \max_{d\in\mathcal{D}} \frac{\textsc{flow}(e,f,d)/c_e}{\textsc{opt}(d)} \qquad \forall e \in E \qquad (7) \\ & f \text{ is a routing.} \end{aligned}$$

Notice that constraint (7) can be written as

$$\max_{d\in\mathcal{D}} (\textsc{flow}(e,f,d) - r c_e \textsc{opt}(d)) \leq 0 \qquad \forall e \in E. \tag{8}$$

4 A model with lower and upper bounds on demands

Suppose that we are given vectors $a = (a_{ij})$ and $b = (b_{ij})$, $(i,j) \in D$, and that $\mathcal{D}$ is the set of all feasible demands d such that $a \leq d \leq b$. For each edge $e \in E$, the left-hand side of (8) is the solution to an optimisation problem over variables d supposing that f and r are fixed. We impose that d is a feasible demand by introducing auxiliary flow variables g. Let us write flow conservation constraints (3) in matricial form $A_1 g = d$ and $A_2 g = \mathbf{0}$; analogously, we use $Bg \leq c$ instead of (4). As $\textsc{flow}(e, f, d) = d^T f_e$, the left-hand side of (8) is the following LP problem in variables g, d and $\omega = \textsc{opt}(d)$, where f_e and r are taken as parameters:

$$
\begin{aligned}
& & \max\ (d^T f_e - r c_e \omega) & \qquad (9)\\
\text{s.t.} \quad & (\pi_e) & A_1 g = d & \qquad (10)\\
& (\sigma_e) & A_2 g = \mathbf{0} & \qquad (11)\\
& (\eta_e) & Bg \leq c\,\omega & \qquad (12)\\
& (\chi_e) & \omega \leq 1 & \qquad (13)\\
& (\lambda_e) & -d \leq -a & \qquad (14)\\
& (\mu_e) & d \leq b & \qquad (15)\\
& & (g, d, \omega) \geq \mathbf{0}, & \qquad (16)
\end{aligned}
$$

where we impose $\omega \leq 1$ to guarantee that the network capacity supports flow g. On the left of each constraint we give the corresponding dual variables. The dual is the minimisation problem

$$
\begin{aligned}
& \min \quad \chi_e - a\lambda_e + b\mu_e & \\
\text{s.t.} \quad & \pi_e^T A_1 + \sigma_e^T A_2 + \eta_e^T B \geq 0 & \qquad (17)\\
& -\pi_e - \lambda_e + \mu_e \geq f_e & \qquad (18)\\
& -c\eta_e + \chi_e \geq -r c_e & \qquad (19)\\
& (\chi_e, \eta_e, \lambda_e, \mu_e) \geq \mathbf{0}. & \qquad (20)
\end{aligned}
$$

Therefore, for each edge $e \in E$ we solve the dual of a maximisation problem that gives the left-hand side of (8).

Proposition 1. *If traffic demands are subject to box constraints, the non-linear constraint (8) is equivalent to (17, 18, 19, 20), and constraint:*

$$\chi_e - a\lambda_e + b\mu_e \leq 0 \qquad \forall e \in E. \qquad (21)$$

Proof. Consider the following optimisation problem and its Lagrangian dual, where non-negative Lagrangian multipliers g, d, ω and z are associated to constraints (17), (18), (19) and (21), respectively.

$$\min\{0 : (17), (18), (19), (20), (21)\} = \max_{(g,d,\omega,z) \geq \mathbf{0}} \mathcal{L}(g, d, \omega, z) =$$

$$
\begin{aligned}
&= \max_{(g,d,\omega,z)\geq \mathbf{0}} \; \min_{(\chi,\eta,\lambda,\mu)\geq \mathbf{0}} \; \big\{0 + g^T(-\pi_e^T A_1 - \sigma_e^T A_2 - \eta_e^T B) + \\
&\qquad\qquad + d^T(\pi_e + \lambda_e - \mu_e + f_e) + \\
&\qquad\qquad + \omega(c\eta_e - \chi_e - rc_e) \\
&\qquad\qquad + z(\chi_e - a\lambda_e + b\mu_e)\big\} =
\end{aligned}
$$

$$
\begin{aligned}
= \max_{(g,d,\omega,z)\geq \mathbf{0}} \big\{ & \, d^T f_e - rc_e\omega + \\
& + \min\{\pi_e^T(-A_1 g + d)\} + \min\{\sigma_e^T(-A_2 g)\} + \\
& + \min_{\chi\geq 0}\{\chi_e(-\omega + z)\} + \min_{\eta\geq 0}\{\eta^T(-Bg + c\omega)\} + \\
& + \min_{\lambda\geq 0}\{\lambda^T(d - za)\} + \min_{\mu\geq 0}\{\mu^T(-d + zb)\}\big\}.
\end{aligned}
$$

This problem has finite solution $d^T f_e - rc_e\omega$ if constraints (10), (11), (12) hold, if $\omega \leq z$, and if $za \leq d \leq zb$. Assuming $z > 0$ (if $z = 0$ we get the null solution), the variable change obtained by dividing g, d, and ω by z gives the original problem (9)-(16). Weak duality then gives (8).

We are now able to give an LP model, which we call MB, to compute in polynomial time $\mathrm{OPR}(\mathcal{D})$, where $\mathcal{D}$ is the set of feasible demands d with box constraints $a \leq d \leq b$.

Proposition 2. *The optimal oblivious routing with box constraints on d is obtained by solving the following linear problem:*

$$
\begin{aligned}
\text{(MB)} \qquad & \min \; r \\
& A_1 f = \mathbf{1} \\
& A_2 f = \mathbf{0} \\
& \chi_e - a\lambda_e + b\mu_e \leq 0 && \forall e \in E \\
& \pi_e^T A_1 + \sigma_e^T A_2 + \eta_e^T B \geq \mathbf{0} && \forall e \in E \\
& -\pi_e - \lambda_e + \mu_e \geq f_e && \forall e \in E \\
& -c\eta_e + \chi_e \geq -rc_e && \forall e \in E \\
& (r, f, \chi, \eta, \lambda, \mu) \geq \mathbf{0}.
\end{aligned}
$$

5 Modelling statistical uncertainty

Suppose that d has a probability distribution (not necessarily known) with mean $\bar{d}$ and covariance matrix P, i.e., $d \in \left\{\bar{d} + Pu : ||u||_2 \leq \epsilon\right\}$, with P positive semidefinite. We replace variable d with $\bar{d} + Pu$ and get a maximisation problem similar to (9)-(16):

$$
\begin{aligned}
& & \max\,(\bar{d} + Pu)^T f_e - T_e c_e \omega \\
\text{s.t.} \quad & (\pi_e) & A_1 g = d \\
& (\sigma_e) & A_2 g = 0 \\
& (\eta_e) & Bg \leq c\,\omega \\
& & \omega \leq 1, \quad d = \bar{d} + Pu, \quad ||u||_2 \leq \epsilon \\
& & (g, d, \omega) \geq \mathbf{0},
\end{aligned}
$$

where we have associated multipliers π_e, σ_e, and η_e to the first three constraints. Consider its Lagrangian dual problem:

$$\min_{\pi_e,\sigma_e,\eta_e\geq 0} \quad \max_{(u,\omega,g)\in S} \quad \{f_e^T(\bar{d}+Pu) - rc_e\omega + \\ +\pi_e^T(\bar{d}+Pu-A_1g)+\sigma_e^T(-A_2g)+\eta_e^T(c\omega-Bg)\}, \tag{22}$$

where $S = \{(u,\omega,g) : ||u||_2 \leq \epsilon, 0 \leq \omega \leq 1, g \geq 0\}$. The inner problem is decomposed in the sum of $f_e^T\bar{d} + \pi_e^T\bar{d} = (f_e+\pi_e)^T\bar{d}$ plus the following three maximisation problems:

a) $\max_{u:||u||_2\leq\epsilon}(f_e+\pi_e)^TPu = \epsilon||P(f_e+\pi_e)||_2$;
b) $\max_{0\leq\omega\leq 1}(\eta_e^Tc - rc_e)\omega = \max(0, \eta_e^Tc - rc_e)$ (we denote it as $[\eta_e^Tc - rc_e]_+$);
c) $\max_{g\geq 0}(-\pi_e^TA_1 - \sigma_e^TA_2 - \eta_e^TB)g$: this problem has null solution if $\pi_e^TA_1 + \sigma_e^TA_2 + \eta_e^TB \leq 0$, otherwise it is unbounded.

Thus, the problem (22) is equivalent to $\min_{\pi_e,\sigma_e,\eta_e\geq 0}(f_e+\pi_e)^T\bar{d}+\epsilon||P(f_e+\pi_e)||_2 + [\eta_e^Tc - rc_e]_+$ provided the condition $\pi_e^TA_1 + \sigma_e^TA_2 + \eta_e^TB \leq 0$ holds. However, the minimization can be eliminated.

Proposition 3. *If the traffic demand d is subject to statistical uncertainty, constraint (8) is equivalent to the following constraints:*

$$(f_e+\pi_e)^T\bar{d}+\epsilon||P(f_e+\pi_e)||_2 + \xi_e \leq 0 \tag{23}$$

$$\xi_e \geq \eta_e^Tc - rc_e \tag{24}$$

$$\pi_e^TA_1 + \sigma_e^TA_2 + \eta_e^TB \geq \mathbf{0} \tag{25}$$

$$(\xi_e, \eta_e) \geq \mathbf{0}. \tag{26}$$

Proof. Consider the optimisation problem $\min\{0 : (23), (24), (25), (26)\}$. Let us take the conic dual to obtain

$$\begin{aligned} & \sup \ \alpha f_e^T\bar{d} + \alpha\epsilon u^TPf_e - \omega rc_e \\ \text{s.t.} \quad & A_1g = \alpha(\bar{d}+\epsilon Pu) \\ & A_2g = \mathbf{0} \\ & \omega \leq \alpha \\ & Bg \leq \omega c \\ & (\alpha, g, \omega) \geq \mathbf{0},\ ||u||_2 \leq 1. \end{aligned}$$

Now let $v = \alpha u$ (therefore $||v||_2 \leq \alpha$) and replace all occurrences of αu by v to obtain the conic dual:

$$\begin{aligned} & z^\star = \sup \ \alpha f_e^T\bar{d} + \epsilon v^TPf_e - \omega rc_e \\ \text{s.t.} \quad & A_1g = \alpha\bar{d}+\epsilon Pv \\ & A_2g = \mathbf{0} \\ & \omega \leq \alpha \\ & Bg \leq \omega c \\ & (\alpha, g, \omega) \geq \mathbf{0},\ ||v||_2 \leq \alpha. \end{aligned}$$

This is a second-order cone programming (SOCP) problem; by weak duality, $z^\star \leq 0$. Now, let $\alpha = 1$ and consider the value $t^\star$ of the objective for this restricted problem: as $t^\star \leq z^\star$, we can get rid of the *min* operator in the constraint

$$\min\{(f_e + \pi_e)^T \bar{d} + \epsilon ||P(f_e + \pi_e)||_2 + \xi_e\} \leq 0,$$

thus proving our result.

Proposition 4. *The model MS for optimal oblivious routing with uncertainty expressed by mean-variance is the following:*

$$
\begin{aligned}
\text{(MS)} \qquad & \min \ r && \\
& A_1 f = \mathbf{1} && \\
& A_2 f = \mathbf{0} && \\
& (f_e - \pi_e)^T \bar{d} + \epsilon ||P(f_e - \pi_e)||_2 + \xi_e \leq 0 && \forall e \in E \qquad (27) \\
& \xi_e \geq \eta_e^T c - r c_e && \forall e \in E \qquad (28) \\
& \pi_e^T A_1 + \sigma_e^T A_2 + \eta_e^T B \geq \mathbf{0} && \forall e \in E \qquad (29) \\
& (r, f, \eta, \xi) \geq \mathbf{0}. &&
\end{aligned}
$$

This model has the non-linear but convex, second-order cone constraint (27) and hence is solvable in polynomial time through interior point SOCP solvers. Notice that constraints (28) and (29) are equivalent to (19) and (17).

6 Computational results

We have adopted a test bed of four network instances available from the Rocketfuel project [13], providing data for the topology (V and E), link counts, and OSPF weights w_e of several real-world networks; we have also used an example instance (*Nsf*) from a work by Mitra et al. [11], with demand and capacity data. We assume that weights follow Cisco's policy that assigns to a link e an OSPF weight w_e equal to the inverse of its capacity. Hence, we simply assume $c_e = 1/w_e$.

The data on the traffic matrix, regarded as proprietary information by Internet Service Providers (ISPs), is rarely disclosed. Therefore, we have created the traffic matrix under the *gravity* model: the demand is assumed proportional to a *repulsion* and an *attraction* parameter, R_i and A_i, associated to each node i, which in turn are proportional to the number of data packets exiting and entering node i, respectively. In order to test our model on a reasonable demand matrix, we use a scalar factor β such that the demands $d_{ij} = \beta R_i A_j$ are feasible. Assume $\beta = \gamma \max\{u : (3), (4), (5), d_{ij} = u R_i A_j\}$ for a given $\gamma \in [0, 1]$. Hence, $(\beta R_i A_j)$ is a feasible traffic matrix with congestion at most equal to γ.

We have tested our instances using different values of the uncertainty parameters. The scalar γ has been assigned values in the set $\{0.75, 0.95, 0.99\}$, so as to give $\bar{d}$ increasingly critical values. In model MB, the lower and upper bounds are $a = \bar{d}/p$ and $b = \bar{d}p$, where p has been assigned values in the set $\{1.2, 2, 5, 20\}$. In model MS, the covariance matrix P is a positive semidefinite, randomly generated matrix. The parameter ϵ is set to $\zeta||\bar{d}||_2$, where $\zeta \in \{0.01, 0.05, 0.1, 0.2, 0.4, 0.8\}$. It is worth noticing that large values of p and ζ correspond to a greater degree of uncertainty, hence we expect the oblivious performance ratio to grow as p and ζ grow.

The size of all instances tested could be reduced by neglecting those nodes in V with degree one, as the routing to and from such nodes is trivial. More precisely, if the removal of an edge $e \in E$ divides the graph in two components G_1 and G_2 such that G_1 is a tree, then $G_1 = (V_1, E_1)$ can be shrunk into a supernode i whose demands d_{ij} (resp. d_{ji}) for all nodes $j \notin V_1$ are given by $\sum_{h \in V_1} d_{hj}$ (resp. $\sum_{h \in V_1} d_{jh}$), while all *internal* demands d_{hk} with both h and k in V_1 can be ignored as they are routed within G_1. A polynomial procedure to reduce the graph consists in repeatedly shrinking all nodes i with only one neighbour j to j itself, until no such nodes i are found. It is worth noting that each the flow on edge $e = \{i, j\}$ is fixed, hence $\text{FLOW}(e, f, d)/c_e \leq \text{OPT}(d)$; as $\text{OPR}(\mathcal{D}) \geq 1 \geq \frac{\text{FLOW}(e,f,d)/c_e}{\text{OPT}(d)}$, edge e can be ignored. As appears from columns 2 to 5 in Tables 1 and 2, this reduces the size of almost all instances, which could be solved to optimality in reasonable time.

	Orig.		Redu.			$\gamma = 0.75$		$\gamma = 0.90$		$\gamma = 0.99$		
Name	$\|V\|$	$\|E\|$	$\|V\|$	$\|E\|$	p	oopr	ospf	oopr	ospf	oopr	ospf	t_{avg} [s]
Telstra (AU)	44	46	7	9	1.2	1.0000	1.9852	1.0000	1.9739	1.0000	1.9694	0.01
					2.0	1.0000	2.0303	1.0000	2.0167	1.0000	2.0140	
					5.0	1.2343	2.0607	1.2343	2.0553	1.2343	2.0542	
					20.0	1.2827	2.0759	1.2827	2.0746	1.2827	2.0743	
VNSL (India)	9	11	8	10	1.2	1.0000	2.1163	1.0000	2.1142	1.0000	2.1135	1.05
					2.0	1.0000	2.1231	1.0000	2.1210	1.0000	2.1206	
					5.0	1.0000	2.1279	1.0000	2.1270	1.0000	2.1269	
					20.0	1.0000	2.1303	1.0000	2.1301	1.0000	2.1300	
Nsf (US)	8	10	8	10	1.2	1.1824	2.0926	1.1824	2.0926	1.1822	2.0926	0.90
					2.0	1.6069	3.0118	1.6069	3.0025	1.6069	2.9605	
					5.0	1.8057	3.6850	1.8040	3.6010	1.8036	3.5842	
					20.0	1.8155	3.9213	1.8148	3.9002	1.8147	3.8961	
Abovenet (US)	19	34	15	30	1.2	1.0000	2.3789	1.0000	2.3507	1.0000	2.3241	229.41
					2.0	1.0000	3.5284	1.0000	3.5284	1.0000	3.5284	
					5.0	1.0000	3.9172	1.0000	3.9124	1.0000	3.9114	
					20.0	1.3440	7.0130	1.3440	7.0130	1.3440	7.0130	
Sprintlink (US)	52	85	33	65	1.2	1.0000	1.4346	1.0000	1.4219	1.0000	1.4010	51495.02
					2.0	1.0000	1.6827	1.0000	1.6201	1.0000	1.6076	
					5.0	1.3703	7.7900	1.3703	7.7900	1.3703	7.7900	
					20.0	1.9901	12.8507	1.9901	12.8507	1.9901	12.8507	

Table 1. Oblivious performance ratio for OSPF and the optimal oblivious routing with box uncertainty.

Name	Orig. $\|V\|$	Orig. $\|E\|$	Redu. $\|V\|$	Redu. $\|E\|$	ζ	$\gamma = 0.75$ oopr	$\gamma = 0.75$ ospf	$\gamma = 0.90$ oopr	$\gamma = 0.90$ ospf	$\gamma = 0.99$ oopr	$\gamma = 0.99$ ospf	t_{avg} [s]
Telstra (AU)	44	46	7	9	0.01	1.0000	2.0810	1.0000	2.0810	1.0000	2.0810	0.06
					0.05	1.0000	2.0810	1.0000	2.0810	1.0000	2.0810	
					0.10	1.0609	2.0810	1.0346	2.0810	1.0278	2.0810	
					0.20	1.1016	2.0810	1.0737	2.0810	1.0680	2.0810	
					0.40	1.1603	2.0810	1.1445	2.0810	1.1413	2.0810	
					0.80	1.2827	2.0810	1.2827	2.0810	1.2827	2.0810	
VNSL (India)	9	11	8	10	0.01	1.0000	2.1311	1.0000	2.1311	1.0000	2.1311	1.84
					0.05	1.0000	2.1311	1.0000	2.1311	1.0000	2.1311	
					0.10	1.0000	2.1311	1.0000	2.1311	1.0000	2.1311	
					0.20	1.0000	2.1311	1.0000	2.1311	1.0000	2.1311	
					0.40	1.0000	2.1311	1.0000	2.1311	1.0000	2.1311	
					0.80	1.0041	2.1311	1.0041	2.1311	1.0041	2.1311	
Nsf (US)	8	10	8	10	0.01	1.0835	2.0147	1.0802	2.0147	1.0792	2.0147	3.28
					0.05	1.2781	2.4483	1.2733	2.4483	1.2716	2.4483	
					0.10	1.4060	2.7815	1.4029	3.7815	1.4019	3.7815	
					0.20	1.5365	3.1327	1.5354	3.1327	1.5351	3.1327	
					0.40	1.6461	3.4622	1.6461	3.4622	1.6461	3.4622	
					0.80	1.7415	3.6602	1.7415	3.6602	1.7415	3.6602	
Abovenet (US)	19	34	15	30	0.01	1.0000	2.5162	1.0000	2.5162	1.0000	2.5162	584.17
					0.05	1.0000	2.8322	1.0000	2.8322	1.0000	2.8322	
					0.10	1.0000	2.9830	1.0000	2.9829	1.0000	2.9829	
					0.20	1.0003	3.9292	1.0000	3.9292	1.0003	3.9293	
					0.40	1.0004	3.9353	1.0002	3.9353	1.0007	3.9353	
					0.80	1.1995	6.2078	1.1993	6.2069	1.1995	6.2066	

Table 2. Oblivious performance ratio for OSPF and the optimal oblivious routing under statistical uncertainty.

We have tested the MB model on a Sun Fire 240 workstation equipped with a 1.66 GHz Sparc64 processor and 4 GB RAM; the MS model instead has been tested on a computer with a 1.5 MHz Pentium processor and 512 MB of RAM memory. Both models for optimal oblivious routing have been coded in AMPL [8]; model MB has been solved by the linear programming solvers of CPLEX 9.0 [5] (we have chosen to use the barrier instead of the simplex method due to a substantial improvement in solution time), whereas the SOCP model MS has been solved through the interior point method in the MOSEK 3.1 software package [1]. The source and the data files used in our tests are available from the ftp page:

`ftp://ftp.elet.polimi.it/users/Pietro.Belotti/oblivious`

Table 1 shows the results obtained with the MB model. Columns 4 and 5 give the size of the instance after the reduction above described, then for each value of γ and p we report the optimal oblivious performance ratio “oopr” and the performance ratio “ospf” obtained by OSPF routing. We obtain “ospf” by simply computing, for each pair (s, t), the shortest path from s to t according to the

OSPF weights $w_e = 1/c_e$, and then fixing the flow variables f in MB accordingly. The last column reports the computing time required, on average, to solve the LP problem associated to MB.

Analogously, Table 2 shows the results obtained with the MS model. For each value of γ and ζ we report the optimal oblivious performance ratio "oopr" and the performance ratio "ospf" obtained by OSPF routing, which is obtained similarly as for MB. Due to its size, instance *Sprintlink* of model MS could not fit into the RAM memory and hence has not been solved.

It is apparent from the tables that, in all cases, OSPF routing has an oblivious performance ratio that is much worse than the optimal oblivious routing, computed through our models. With low degrees of uncertainty, while OSPF routing has a sensible performance loss (from 40% for *Sprintlink* network, under the MB model, to 151% for *Abovenet*, under the MS model), the optimal oblivious performance ratio is one in most cases, as is expected since, for $p \to 1$ or $\zeta \to 0$, the optimal routing is the one obtained with model (1)-(5). Nevertheless, $\text{OPR}(\mathcal{D}) = 1$ even for larger p and ζ, i.e., even greater degrees of uncertainty do not affect the routing performance.

As p and ζ get large values, OSPF routing has a performance ratio of up to 12 as in instance *Sprintlink*, whereas the best oblivious routing does not worsen significantly, as the performance loss is not greater than 99%, indicating high robustness of optimal oblivious; notice that $p = 20$ and $\zeta = 0.8$ give a large percentage of the feasible demands. We also observe that the performance ratio has almost no dependence on γ, which can be explained by the fact that γ specifies how critical the demand is w.r.t. the network capacity, but it does not drive the level of uncertainty, which is specified by p and ζ.

We have depicted the dramatic gain in performance in Figure 1 for network *Nsf*, under the MS model, for $\gamma = 0.95$ and for ζ varying in the interval $[10^{-5}, 5]$. It is worth emphasising that the optimal performance ratio is 1 for small and medium values of ζ, whereas the OSPF routing has a performance ratio of 1.6, i.e., a loss of 60%, even for very low degrees of uncertainty. For higher degrees of uncertainty, the OSPF routing attains a performance ratio of 4 while the optimal oblivious ratio stabilises at 1.818. This shows that a finely tuned routing, at least for low degrees of uncertainty, may have a performance ratio which is the best possible, and does not increase significantly even with high uncertainty.

The optimisation time is reasonably short for all instances except *Sprintlink*, that has a size almost double as that of the remaining ones and has required greater memory and processor resources. For larger networks it could be necessary to study an alternative approach, e.g., a column generation technique based on a path formulation of the problem.

7 Concluding remarks

We have proposed two models inspired by the robust optimization methodology of [4] to obtain a routing with optimal oblivious performance with respect to two models of demand uncertainty. The first is a linear programming model that deals

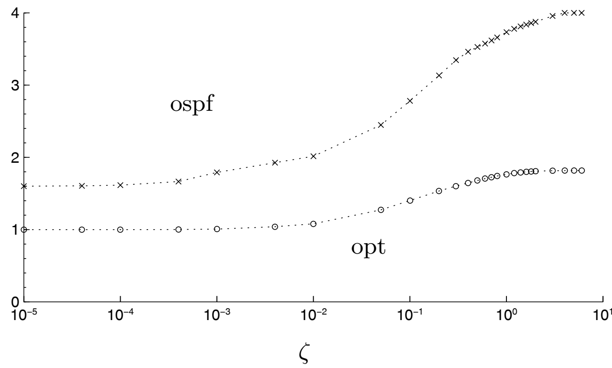

Fig. 1. Comparison of the OSPF and the optimal oblivious performance ratio for network *Nsf* with $\gamma = 0.95$ and different values of ζ.

with demands whose uncertainty is modelled by box constraints. In order to deal with statistical uncertainty, we have proposed a second-order cone programming model to obtain the optimal routing given a mean-covariance representation of the traffic matrix. This proves that the problem of finding optimal oblivious routing with box or statistical uncertainty can be solved in polynomial time.

From a more practical viewpoint, we compare the optimal oblivious routing with the more common OSPF routing technique, where edge weights are fixed according to a simple rule. We have observed that an optimised routing has a much better performance ratio, and a good level of robustness even with high uncertainty. It remains to be investigated whether a better choice of OSPF weights can improve the performance observed in our tests.

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