An Airline Revenue Management Problem

Background: Deregulation occurred in 1978
Prior to Deregulation
- Carriers only allowed to fly certain routes. Hence airlines such as Northwest, Eastern, Southwest, etc.
- Fares determined by Civil Aeronautics Board (CAB) based on mileage and other costs — (CAB no longer exists)
Post Deregulation
- Any carrier can fly anywhere
- Fares determined by carrier (and the market)

Special Features of Airline Economics

- Huge sunk and fixed costs
  - Purchase of airplanes
  - Gate facilities
  - Fuel and crew costs
- Low variable costs per passenger
  - $10/passenger or less on most flights
- Strong economically competitive environment
  - Near-perfect information and negligible cost of information
  - Symmetric information
- No inventories of “product”
  - An empty seat has lost revenue forever: highly perishable inventory.

Multiple fare classes: a monopolist’s perspective

The two fare model presumes that customers are willing to pay the higher price, even if the lower price is available. How did airlines achieve this?

Two Complexities in Revenue Management

- Complexities due to use of hubs.
  - Many customers transfer airplanes at a hub
  - Hubs permit many more “itineraries” to be flown
- Complexities due to uncertainties
  - Typically the less expensive Q fares are sold in advance of the more expensive Y fares.
  - How many tickets should be reserved for Y fares
- Today: We will focus on the complexities due to hubs, and will consider a very simple example.

Four Flights from East-West Airlines

<table>
<thead>
<tr>
<th>Flight #</th>
<th>Depart</th>
<th>Arrive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>Boston</td>
<td>Chicago</td>
</tr>
<tr>
<td>1B</td>
<td>Chicago</td>
<td>San Francisco</td>
</tr>
<tr>
<td>2A</td>
<td>New York</td>
<td>Chicago</td>
</tr>
<tr>
<td>2B</td>
<td>Chicago</td>
<td>Los Angeles</td>
</tr>
</tbody>
</table>

Both planes have a seating capacity of 200

Several passenger itineraries can be determined from these flights. For example, a passenger can fly from Boston to Chicago, and another passenger can fly from Boston to LA.
A Diagram Showing the East-West Flights

Fares and Demand for Itineraries

<table>
<thead>
<tr>
<th>Itinerary</th>
<th>Q-class fare and demand</th>
<th>Y-class fare and demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-C</td>
<td>$200 25</td>
<td>$230 20</td>
</tr>
<tr>
<td>B-SF</td>
<td>$320 55</td>
<td>$420 40</td>
</tr>
<tr>
<td>B-LA</td>
<td>$400 65</td>
<td>$490 25</td>
</tr>
<tr>
<td>NY-C</td>
<td>$250 24</td>
<td>$290 16</td>
</tr>
<tr>
<td>NY-SF</td>
<td>$410 65</td>
<td>$550 50</td>
</tr>
<tr>
<td>NY-LA</td>
<td>$450 40</td>
<td>$550 35</td>
</tr>
<tr>
<td>C-SF</td>
<td>$200 21</td>
<td>$230 20</td>
</tr>
<tr>
<td>C-LA</td>
<td>$250 25</td>
<td>$300 14</td>
</tr>
</tbody>
</table>

Number of seats allocated if everyone flies

<table>
<thead>
<tr>
<th></th>
<th>Q-demand</th>
<th>Y-demand</th>
<th>Seat capacity: 200 per flight</th>
<th>Y-fares are higher</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF</td>
<td>141, 110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NY</td>
<td>130, 74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>129, 101</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>145, 85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Formulation as a Linear Program

- What are the decision variables?
- What is the objective?
- What are the constraints?

An Abstracted version of the LP

- Let $F$ be the set of flights
- Let $C$ be the set of itineraries/classes
  - e.g., $<\text{NY-C-SF 7:45-12:15, Q-class}> \in C$
- $r_j = \text{revenue from } j \in C$
- $d_j = \text{demand for } j \in C$
- let $C(f)$ = subset of $C$ containing flight $f$
- $c_f = \text{capacity of flight } f$

Work with your partner to formulate the LP

The Optimal Solution

<table>
<thead>
<tr>
<th>Itinerary</th>
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<th>Y-class sold and demand</th>
</tr>
</thead>
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<tr>
<td>B-C</td>
<td>25 25</td>
<td>20 20</td>
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<td>B-SF</td>
<td>25 55</td>
<td>40 40</td>
</tr>
<tr>
<td>B-LA</td>
<td>65 65</td>
<td>25 25</td>
</tr>
<tr>
<td>NY-C</td>
<td>19 24</td>
<td>16 16</td>
</tr>
<tr>
<td>NY-SF</td>
<td>44 65</td>
<td>50 50</td>
</tr>
<tr>
<td>NY-LA</td>
<td>36 40</td>
<td>35 35</td>
</tr>
<tr>
<td>C-SF</td>
<td>21 21</td>
<td>20 20</td>
</tr>
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<td>14 14</td>
</tr>
</tbody>
</table>
Robert L. Crandall, Chairman, President, and CEO of AMR

I believe that yield management is the single most important technical development in transportation management since we entered the era of airline deregulation in 1979.

The development of American Airline's yield-management system has been long and sometimes difficult, but this investment has paid off. We estimate that yield management has generated $1.4 billion in incremental revenue in the last three years alone. This is not a one-time benefit. We expect it to generate at least $500 million annually for the foreseeable future.

Math Programming and Radiation Therapy

- Based on notes developed by Rob Freund (with help from Peng Sun)
- Lecture notes from 15.094
In conventional radiotherapy:
- 3 to 7 beams of radiation
- Radiation oncologist and physicist work together to determine a set of beam angles and beam intensities
- Determined by manual “trial-and-error” process

With only a small number of beams, it is difficult/impossible to deliver required dose to tumor without impacting the critical area.

- More accurate map of tumor area
- CT — Computed Tomography
- MRI — Magnetic Resonance Imaging
- More accurate delivery of radiation
- IMRT: Intensity Modulated Radiation Therapy
- Tomotherapy

- For a given tumor and given critical areas
- For a given set of possible beamlet origins and angles
- Determine the weight on each beamlet such that:
  - Dosage over the tumor area will be at least a target level \( \gamma_t \)
  - Dosage over the critical area will be at most a target level \( \gamma_c \)
Opportunities for enhancements

- Use penalties: e.g., $D_{ij} \geq \gamma - y_{ij}$ and then penalize $y$ in the objective.
- Consider non-linear penalties (e.g., quadratic)
- Consider costs that depend on damage rather than on radiation
- Develop target doses and penalize deviation from the target
Summary

- Revenue management, tomotherapy
- Models are rarely perfect. One balances the quality of the model with the needs for the situation.
- Some techniques used: penalties, reformulations.