You will find below eight sample questions for the IE444 midterm examination. I cannot guarantee that the questions in the actual exam will be identical to those below. I designed these questions to give an idea of the level of difficulty you should expect. I will not distribute the answers to these questions. You should be able to gather the right answers yourself, possibly with the help of fellow classmates. The exam will be conducted open notes. You are only allowed to bring your classroom notes and hand-outs. Do not bring the answers to the questions below with you to the exam.

1. State whether true or false:
   A. A call option is a contract that gives you the right to sell an asset at a specific price within a specific time period.
   B. An American call can only be exercised at expiration date.
   C. Markowitz’ Mean-Variance portfolio optimization problem is a linear programming problem.
   D. Consider a contract that gives you the right to buy an asset a month from now. The asset is trading at $100 dollars per share now. If the contract is sold now at $95, you should buy this contract.
   E. The Value-at-Risk measure gives the expected loss conditioned on the event that the maximum loss will exceed some amount.
   F. If a $100 par-valued zero-coupon bond has price $B$ currently, then $B \geq 100$.
   G. At a fixed percentile, CVaR is always less than VaR.
   H. VaR is a subadditive risk measure.
   I. Consider a European put option, with strike price $100. This option is worthless at maturity if the price of the asset is greater than $100.
   J. Consider a European put option, with strike price $100. This option is valuable at maturity if the price of the asset is less than $100.

2. Mr W. sells Mrs B. the right to buy 100 shares at $50 per share a month from
now. With probability 1/2 the share price can go up to $70, a month from now. Or, it can go down to $30 with probability 1/2. The share currently trades at $45 and the risk-free rate is 12% p.a. compounded monthly. Mrs B. values this contract $1000. Is she correct? In case your answer is negative, advise Mrs B. on the right price. Explain your answer.

3. Suppose that there are two assets with mean return 0.12 and .15, respectively. The variance of the returns are given by 0.20 and 0.18 along with the covariance equal to 0.01. For a portfolio with weights \( w_1 = 0.25 \) and \( w_2 = 0.75 \), calculate the mean return and the volatility (variance of the portfolio).

4. Assume that a pay-off matrix \( A \) and price vector \( p \) is given for some assets and scenarios. Is it true that if there is no arbitrage, then any two portfolios having the same pattern of pay-offs will have the same price?

5. Suppose that in the Markowitz’ mean-variance portfolio model with shorting allowed, there are three uncorrelated assets (their covariances are zero). Each has variance 1, and the mean return values are 2, 4, and 6, respectively. Find the minimum variance point of the efficient frontier corresponding to these assets.

6. Suppose that in the Markowitz’ mean-variance portfolio model, there are three uncorrelated assets (their covariances are zero). Each has variance 1, and the mean return values are 3, 4, and 6, respectively. Assume no shorting is allowed. Find an efficient portfolio.

7. Suppose that the loss function for a certain investment is given by \( f(x, y) = -y \) where \( y = 100 - j \), for \( j = 1, \ldots, 99 \) with probability 0.01 and \( y = -500 \) with probability 0.01. Calculate the 0.95 VaR and CVaR for this investment.

8. Suppose there are \( n \) assets which are uncorrelated. You may invest in any one, or any combination of them. The mean rate of return \( \bar{r} \) is the same for each asset, but the variances are different. The return on asset \( i \) has a variance \( \sigma_i^2 \) for \( i = 1, \ldots, n \). Find the minimum variance point. Express your result in terms of

\[
\bar{\sigma}^2 = \left( \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \right)^{-1}.
\]